

#### TAB: BOND PARAMETERS

Bond terms		
General parameters		
Parameter	Value	Description
T	10.00	Term to maturity
dt	2.00	Time increment
principal	100.0	Bond principal amount
coupon-rate	5.0%	Coupon / dividend rate
coupon-frequency	1	Coupon payment frequency (measured in the # of tree periods)
n	6	Number of periods in the tree that measure the bond life
Interest deferral		
Parameter	Value	Description
interest deferral period	0	The latest period until the bond coupon payments are deferred
is compounded	1	1 if deferred interest is compounded and zero otherwise
Bond amortization schedule		
Parameter	Value	Description
1	0%	% of bond amortization in period t=0
3	0%	% of bond amortization in period t=3
5	100%	% of bond amortization at maturity
amortization schedule	<code>java.util.LinkedHas</code>	Bond amortization schedule
Bond option parameters		
Parameter	Value	Description
option	call	Option type
Bond redemption premium / discount		
Parameter	Value	Description
1	3.00%	Bond redemption premium in period t=0
2	2.00%	Bond redemption premium in period t=3
3	1.00%	Bond redemption premium at maturity
redemption premium	<code>java.util.LinkedHas</code>	Premium at which the bond is redeemed if option is exercised

TAB: INTEREST RATE TREE PARAMETERS

Interest rate tree and tree calculator parameters			Transition probability functions		
General parameters			Function arguments		
Parameter	Value	Description	Parameter	Value	Description
<i>Input parameters</i>					
$y_0$	5.0%	Initial yield rate	<i>Diffusion drift and volatility function arguments</i>		
$\mu$	0.0%	Drift parameter	$t$	0.0	Time period
$\sigma$	1.07%	Standard deviation	$y$	0.0	Process state action
<i>Derived parameters</i>					
$(\mu - \sigma^2/2) \times dt$	0.0%	Adjusted one-period drift parameter	$x$	0.0	Process state
$\sigma \times \sqrt{dt}$	1.5%	Size of upward / downward movement	$mu$	0.00%	Diffusion drift parameter
<i>Tree parameters</i>					
Parameter	Value	Description	$sigma$	1.07%	Diffusion volatility parameter
<i>States</i>					
$dx_{down}$	4.00	Normalized downward change in state over $dt$ period ( $x_{min} = E x_T - \sum_T x dx_{down}$ )	<i>Transition probability functions</i>		
$dx_{up}$	7.00	Normalized upward change in state over $dt$ period ( $x_{max} = E x_T + \sum_T x dx_{up}$ )	<i>Diffusion drift and volatility functions (used to construct transition probabilities function)</i>		
X-min	-0.30	Lower boundary on minimum state value	jkr.parser.lib.jmc.calculator.Calculator@Sun, 24 Jun 2018 12:54:38:608..113443318301897685.		
X-max	0.30	Upper boundary on maximum state value	'mux' = 'mu';		
X-count	13	Number of process discrete states ( $x_i$ )	'sigmax' = 'sigma';		
iter-count	10	Number of iterations used to construct $\sigma(T, x)$ - uniform set of states in period T	'Fmu' = FX('mux', VARS('t', 'y', 'x'));		
<i>Diffusion tree parameter</i>					
<i>Calculator parameters</i>					
Parameter	Value	Description	'Fsigma' = FX('sigmax', VARS('t', 'y', 'x'));		
<i>Calculator option parameters</i>			<i>Variable name</i>		
option	call	Evaluated option	Fmu	jkr.parser.lib.jmc.formu	Diffusion drift function object
yield-min	-1%	Minimum yield bound in yield binary search estimation	Fsigma	jkr.parser.lib.jmc.formu	Diffusion dvolatility function object
yield-max	20%	Maximum yield bound in yield binary search estimation	<i>Transition probability tree structure</i>		
yield-iter	10	Maximum number of yield iterations	Movement direction $a_i$ $p_i$ Restrictions on the mapping coefficients		
npv-delta	0.1	Maximum difference between bond cash flow yield-based NPV and bond clean price Q tree structure	up	1.732	0.17 $\sum a_i^2 \times p_i = 1$
<i>Calculator option executed methods</i>			zero	0.000	0.67 $\sum p_i = 1$
value-option		Estimate the bond bullet prices and prices in the presence of the option	down	-1.732	0.17
bond-yields		Estimate the bond bullet yields and yields in the presence of the option	java.util.LinkedHashMap		
<i>Parameter objects</i>					
Parameter	Value	Description			
<i>Parameter mappings objects</i>					
States mapping	java.util.Link	State mapping object			
Calculator option parameter mapping	java.util.Link	Calculator parameter mapping object			
Calculator option executed methods mapping	java.util.Arra	Calculator option executed methods list object			

TAB: INTEREST RATE TREE MODELING

Controlled diffusion tree inputs and objects		Controlled diffusion tree output					
Interest rate tree objects		Tree states					
Diffusion tree		Tree states					
Parameter	Value	#					
T	10						
dt	2.00	#					
Y <sub>0</sub>	5.00%						
states	java.util.LinkedHashMap@Sun, 24 Jun 2018 1						
process-params							
Fmu	jkr.parser.lib.jmc.formula.objects.function.Coi						
Fsigma	jkr.parser.lib.jmc.formula.objects.function.Coi						
Q-tree structure	java.util.LinkedHashMap@Sun, 24 Jun 2018 1						
parameters							
jeconkr.finance.HW.Derivatives2003.lib.ch23_srm.ShortRateModel@St							
Tree states		time period					
		#					
		0	1	2	3	4	5
1	state	-9.4%					-9.43%
2		-6.6%					-6.57% -6.57%
3		-3.7%					-3.70% -3.70% -3.70%
4		-0.8%					-0.84% -0.84% -0.84% -0.84%
5		2.0%					2.02% 2.02% 2.02% 2.02% 2.02%
6		4.9%					4.89% 4.89% 4.89% 4.89% 4.89%
7		7.7%					7.75% 7.75% 7.75% 7.75% 7.75%
8		10.6%					10.61% 10.61% 10.61% 10.61% 10.61%
9		13.5%					13.47% 13.47% 13.47% 13.47%
10		16.3%					16.34% 16.34%
11		19.2%					19.20%
12		22.1%					
13		24.9%					
Tree output parameters		Tree state distribution					
Transition probabilities		#					
		#					
		0	1	2	3	4	5
1	t	1	2	3	4	5	6
					x <sub>t</sub>		
						0:-0.008 0:0.02 0:0.049 0:0.077	
1		0:0.02					0.140
2		0:0.049					0.721 0.140
3		0:0.077					0.140 0.721
4		0:0.106					0.140
5		0:0.135					
6	x <sub>t+1</sub>	1:-0.001					1.000 1.000
Distribution of terminal states		Distribution of terminal states					
Calculator and tree parameter mapping							
key	value	mean	4.89%				
t	2	variance	0.114%				
path-avg		standard deviation	3.38%				
path-stdev							
path-pct	java.util.ArrayList@Sun, 24 Jun 2018 12:54:35:7						
Lower percentile	0.25						
Upper percentile	0.75						
Calculator parameters mapping							
Transition probability distribution							

TAB: INTEREST RATE TREE CALCULATOR OUTPUT

## Interest rate tree output

## Interest rate tree calculator objects

Markov tree calculator		Value
Parameter		
Interest rate tree model	jeconkr.finance.HW.Derivatives2003.lib.ch23_srm.Sho	
bond cash flows	java.util.LinkedHashMap@Sun, 24 Jun 2018 13:22:19:	
parameters	java.util.LinkedHashMap@Sun, 24 Jun 2018 12:54:35:	
executed methods	java.util.ArrayList@Sun, 24 Jun 2018 12:54:35:739..64	
jeconkr.finance.HW.Derivatives2003.lib.ch23_srm.calculator.CalculatorSRM@Sun, 2		

## Bond value in the presence of the option (full value including accrued interest)

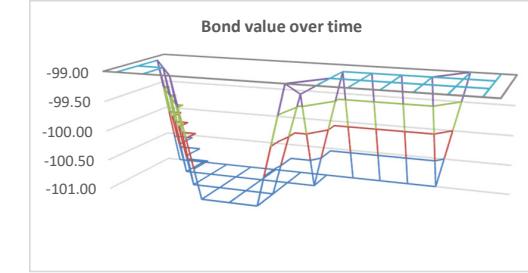
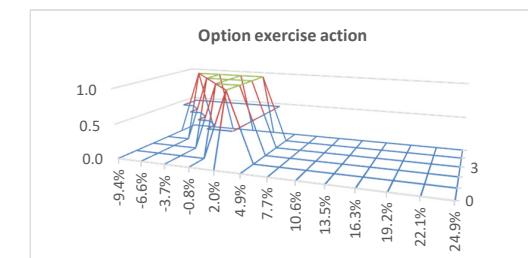
#	state	time period					
		0	1	2	3	4	
1	-9.4%					-110.00	
2	-6.6%				-111.00	-110.00	
3	-3.7%			-111.00	-111.00	-110.00	
4	-0.8%		-112.00	-111.00	-111.00	-110.00	
5	2.0%	-113.00	-112.00	-111.00	-111.00	-110.00	
6	4.9%	<b>-95.67</b>	<b>-106.74</b>	<b>-107.88</b>	<b>-109.00</b>	<b>-109.76</b>	<b>-110.00</b>
7	7.7%	-91.52	-95.09	-99.29	-104.21	-110.00	
8	10.6%		-82.96	-98.97	-110.00		
9	13.5%			-81.83	-94.01	-110.00	
10	16.3%				-89.34	-110.00	
11	19.2%					-110.00	
12	22.1%						
13	24.9%						

## Option exercise action function

#	state	time period				
		0	1	2	3	4
1	-9.4%					0
2	-6.6%				1	0
3	-3.7%			1	1	0
4	-0.8%		1	1	1	0
5	2.0%	1	1	1	1	0
6	4.9%	0	0	0	0	0
7	7.7%	0	0	0	0	0
8	10.6%	0	0	0	0	0
9	13.5%	0	0	0	0	0
10	16.3%	0	0	0	0	0
11	19.2%	0	0	0	0	0
12	22.1%	0	0	0	0	0
13	24.9%					

## Tree calculator output parameters

Tree keys:	Description
field	Markov tree calculator field
bond-price	The full (dirty) price of the bond
bond-price-bullet	The full (dirty) bullet price of the bond (option-adjusted)
bond-ytm	The bond yield to maturity (based on bond clean price)
bond-ytm-bullet	The bond bullet yield to maturity
mode-action	Optimal mode action estimated for the tree
objective	Objective function defined on tree states
discount	Discount function defined on tree states



## Bond bullet value (full value including accrued interest)

#	state	time period					
		0	1	2	3	4	
1	-9.4%					-110.00	
2	-6.6%				-135.44	-110.00	
3	-3.7%			-128.46	-110.00		
4	-0.8%		-146.47	-133.98	-121.87	-110.00	
5	2.0%	-131.88	-126.49	-121.10	-115.64	-110.00	
6	4.9%	<b>-100.01</b>	<b>-109.67</b>	<b>-109.54</b>	<b>-109.59</b>	<b>-109.76</b>	<b>-110.00</b>
7	7.7%	-91.76	-95.16	-99.29	-104.21	-110.00	
8	10.6%		-82.96	-90.07	-98.97	-110.00	
9	13.5%			-81.83	-94.01	-110.00	
10	16.3%				-89.34	-110.00	
11	19.2%					-110.00	
12	22.1%						
13	24.9%						

## Option value

#	state	time period				
		0	1	2	3	4
1	-9.4%					0.000
2	-6.6%				24.439	0.000
3	-3.7%				37.394	17.458
4	-0.8%			34.474	22.985	10.866
5	2.0%		18.880	14.486	10.103	4.640
6	4.9%	<b>4.338</b>	<b>2.931</b>	<b>1.664</b>	<b>0.588</b>	<b>0.000</b>
7	7.7%	0.242	0.070	0.000	0.000	0.000
8	10.6%		0.000	0.000	0.000	0.000
9	13.5%			0.000	0.000	0.000
10	16.3%				0.000	0.000
11	19.2%					0.000
12	22.1%					
13	24.9%					

## Objective function (bond cash flow or bond redemption price)

#	state	time period				
		0	1	2	3	4
1	-9.4%					-220.00
2	-6.6%				-111.00	-220.00
3	-3.7%			-111.00	-111.00	-220.00
4	-0.8%		-112.00	-111.00	-111.00	-220.00
5	2.0%	-113.00	-112.00	-111.00	-111.00	-220.00
6	4.9%	<b>0.00</b>	<b>-10.00</b>	<b>-10.00</b>	<b>-10.00</b>	<b>-220.00</b>
7	7.7%	-10.00	-10.00	-10.00	-220.00	
8	10.6%		-10.00	-10.00	-220.00	
9	13.5%			-10.00	-220.00	
10	16.3%				-220.00	
11	19.2%					
12	22.1%					
13	24.9%					

## Discount function

#	state	time period				
		0	1	2	3	4
1	-9.4%					1.140
2	-6.6%				1.077	1.077
3	-3.7%			1.017	1.017	1.017
4	-0.8%		0.960	0.960	0.960	0.960
5	2.0%	<b>0.907</b>	<b>0.907</b>	<b>0.907</b>	<b>0.907</b>	<b>0.907</b>
6	4.9%	0.856	0.856	0.856	0.856	0.856
7	7.7%	0.809	0.809	0.809	0.809	0.809
8	10.6%		0.764	0.764	0.764	0.764
9	13.5%			0.721	0.721	0.721
10	16.3%					
11	19.2%					
12	22.1%					
13	24.9%					

TAB: BOND YIELD RATES AND OPTION PREMIUM

Parameters		Bond value in the presence of the option						Option annual premium									
		#		time period						#		time period					
principal	100			0	1	2	3	4	5			0	1	2	3	4	5
T	10.00	1	-9.4%							1	-9.4%						
coupon-rate	5.0%	2	-6.6%							2	-6.6%						
coupon-frequency	0	3	-3.7%							3	-3.7%						
dt	2.00	4	-0.8%							4	-0.8%						
		5	2.0%							5	2.0%						
		6	4.9%	5.59%	5.52%	5.43%	5.29%	5.13%		6	4.9%	0.58%	0.47%	0.34%	0.16%	0.00%	
		7	7.7%		8.34%	8.35%	8.36%	8.39%		7	7.7%	0.05%	0.01%	0.00%	0.00%		
		8	10.6%			9.99%	9.99%	9.99%		8	10.6%		0.00%	0.00%	0.00%		
		9	13.5%				9.99%	9.99%		9	13.5%			0.00%	0.00%		
		10	16.3%					9.99%		10	16.3%						
		11	19.2%							11	19.2%						
		12	22.1%							12	22.1%						
		13	24.9%							13	24.9%						

Bond bullet value		Option premium / discount term structure						time period					time period							
#		#						0	1	2	3	4			0	1	2	3	4	5
								0	1	2	3	4			0	1	2	3	4	5
1	-9.4%	1	-9.4%										1	-0.49%						
2	-6.6%	2	-6.6%										2	-0.49%						
3	-3.7%	3	-3.7%										3	-0.49%						
4	-0.8%	4	-0.8%										4	-0.49%						
5	2.0%	5	2.0%										5	2.07%						
6	4.9%	6	4.9%	5.00%	5.05%	5.09%	5.12%	5.13%					6	5.13%						
7	7.7%	7	7.7%		8.29%	8.34%	8.36%	8.39%					7	8.39%						
8	10.6%	8	10.6%			9.99%	9.99%	9.99%					8	9.99%						
9	13.5%	9	13.5%				9.99%	9.99%					9	9.99%						
10	16.3%	10	16.3%					9.99%					10	9.99%						
11	19.2%	11	19.2%										11							
12	22.1%	12	22.1%										12							
13	24.9%	13	24.9%										13							

## TAB: INTEREST TREE STATISTICS

Parameters		Interest rate tree statistics								
Interest rate tree	jeconkr.finance.H	Statistics	Parameter	time period						
Tree output parameters				0	1	2	3	4	5	
Field key	path-stats			mean	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Parameters	java.util.LinkedH			stdev	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Q output parameters				lower quartile	0.25	0.00%	0.00%	0.00%	0.00%	0.00%
Q drift field parameter	Q-drift	upper quartile	0.75	0.00%	0.00%	0.00%	0.00%	0.00%		
Q sigma field parameter	Q-sigma									

The figure displays a line plot of option premium over time. The x-axis represents time from 0 to 5, and the y-axis represents option premium, with a specific point at 1. The plot includes several statistical measures: mean (black lines), lower quartile (red line at y=1), upper quartile (green line at y=1.5), and standard deviation (purple line at y=0).

## Interest rate transitional probability statistics

### *Interest process drift parameter*

#	state	time period				
		0	1	2	3	4
1	-9.4%	0.00%	0.00%	0.00%	0.00%	0.00%
2	-6.6%	0.00%	0.00%	0.00%	0.00%	0.00%
3	-3.7%	0.00%	0.00%	0.00%	0.00%	0.00%
4	-0.8%	0.00%	0.00%	0.00%	0.00%	0.00%
5	2.0%	0.00%	0.00%	0.00%	0.00%	0.00%
6	4.9%	0.00%	0.00%	0.00%	0.00%	0.00%
7	7.7%	0.00%	0.00%	0.00%	0.00%	0.00%
8	10.6%	0.00%	0.00%	0.00%	0.00%	0.00%
9	13.5%	0.00%	0.00%	0.00%	0.00%	0.00%
10	16.3%	0.00%	0.00%	0.00%	0.00%	0.00%
11	19.2%	0.00%	0.00%	0.00%	0.00%	0.00%
12	22.1%	0.00%	0.00%	0.00%	0.00%	0.00%
13	24.9%	0.00%	0.00%	0.00%	0.00%	0.00%

### *Interest process volatility parameter*

#	state	time period				
		0	1	2	3	4
1	-9.4%	0.00%	0.00%	0.00%	0.00%	0.00%
2	-6.6%	0.00%	0.00%	0.00%	0.00%	0.00%
3	-3.7%	0.00%	0.00%	0.00%	0.00%	0.00%
4	-0.8%	0.00%	0.00%	0.00%	0.00%	0.00%
5	2.0%	0.00%	0.00%	0.00%	0.00%	0.00%
6	4.9%	0.00%	0.00%	0.00%	0.00%	0.00%
7	7.7%	0.00%	0.00%	0.00%	0.00%	0.00%
8	10.6%	0.00%	0.00%	0.00%	0.00%	0.00%
9	13.5%	0.00%	0.00%	0.00%	0.00%	0.00%
10	16.3%	0.00%	0.00%	0.00%	0.00%	0.00%
11	19.2%	0.00%	0.00%	0.00%	0.00%	0.00%
12	22.1%	0.00%	0.00%	0.00%	0.00%	0.00%
13	24.9%	0.00%	0.00%	0.00%	0.00%	0.00%

## TAB: BOND INTEREST AND PRINCIPAL PAYMENTS CASH FLOWS

TAB: TESTING CODE EXECUTION

**Parameter testing**

Testing mapping and list input parameters		Testing function input parameters					Testing backward recursion						
State mapping		Probability transition function Q(x)					Tested bond value function					Discount rates	
key		# t y x FT(x)					# 5 4 3 2 1 0					4 3 2 1 0	
java.lang.ClassCastException	java.lang.ClassCastException	1	0	0	-2	#NAME?	1	-9.43%	-110.00			1.140	
java.lang.ClassCastException	java.lang.ClassCastException	2	1	0	-1	#NAME?	2	-6.57%	-110.00	-135.44		1.077	
java.lang.ClassCastException	java.lang.ClassCastException	3	1	0	0	#NAME?	3	-3.70%	-110.00	-128.46		1.017	
java.lang.ClassCastException	java.util.Map	4	0	0	1	#NAME?	4	-0.84%	-110.00	-121.87		0.960	
java.lang.ClassCastException	java.lang.ClassCastException	5	0	0	2	#NAME?	5	2.02%	-110.00	-115.64		0.907	
java.lang.ClassCastException	java.lang.ClassCastException	6	0	0	3	#NAME?	6	4.89%	-110.00	-109.76		0.856	
java.lang.ClassCastException	java.lang.ClassCastException	7	0	0	4	#NAME?	7	7.75%	-110.00	-104.21		0.809	
Calculator methods list object							8	10.61%	-110.00	-98.97		0.764	
value-option							9	13.47%	-110.00	-94.01		0.721	
Test of state uniform distribution							10	16.34%	-110.00	-89.34			
							11	19.20%	-110.00				
							12	22.06%	-110.00				
							13	24.93%	-110.00				
State		Change in state					bond cash flow					Difference between calculated and tested values	
-0.094		$\Sigma$					bond cash flow	110.00	10.00	10.00	10.00	0.00	
-0.066	2.86%	$\mu_T$					bond redemption value	111.00	111.00	111.00	112.00	113.00	100.00
-0.037	2.86%	$x_0$					# 5 4 3 2 1 0					4 3 2 1 0	
-0.008	2.86%	$x_{min}$					1	-9.43%	-110.00			0.000	
0.020	2.86%	$x_{max}$					2	-6.57%	-110.00	-111.00		0.000	
0.049	2.86%						3	-3.70%	-110.00	-111.00		0.000	
0.077	2.86%						4	-0.84%	-110.00	-111.00		0.000	
0.106	2.86%						5	2.02%	-110.00	-111.00		0.000	
0.135	2.86%						6	4.89%	-110.00	-109.76		0.000	
0.163	2.86%						7	7.75%	-110.00	-104.21		0.000	
0.192	2.86%						8	10.61%	-110.00	-98.97		0.000	
0.221	14.91%						9	13.47%	-110.00	-94.01		0.000	
0.249	12.98%						10	16.34%	-110.00	-89.34		0.000	
							11	19.20%	-110.00				
							12	22.06%	-110.00				
							13	24.93%	-110.00				
							dt 2.00						
							# 5 4 3 2 1 0					4 3 2 1 0	
							1	-9.43%				1.151	
							2	-6.57%				1.080	
							3	-3.70%				1.017	
							4	-0.84%				0.961	
							5	2.02%				0.911	
							6	4.89%				0.866	
							7	7.75%				0.825	
							8	10.61%				0.788	
							9	13.47%				0.754	
							10	16.34%					
							11	19.20%					
							12	22.06%					
							13	24.93%					

**TAB: CONFIGURATION FILE**

**Calculation enabled**

**TRUE**

Test connection

host: home-pc; user: Konstantin Rybakov; key: null

#### Output array sizes

Maximum size

Object	number of rows	number of cols
Tree dimension	13	6
Transition probabilities	12	7
T	10.00	
dt	2.00	
n <sub>T</sub>	6	
X-count	13	

Array formula cell address	number of rows	number of cols	Description
'tree'!\$H\$10	13	6	Tree states
'tree'!\$G\$10	13	1	Tree list of states
'tree'!\$R\$10	13	6	Tree probabilities
'tree'!\$G\$28	6	5	Transition probabilities
'calc'!\$H\$10	13	6	Option value function
'calc'!\$R\$10	13	6	Option exercise action value
'yield'!\$H\$8	13	5	Yields
'yield'!\$H\$26	13	5	Bullet yields
'calc'!\$H\$46	13	6	Asset cash flows
'calc'!\$R\$46	13	5	Discount function
'stats'!\$H\$8	4	6	Interest tree statistics (mean, stdev, lower and upper quartiles)
'stats'!\$H\$27	13	5	Interest tree drift parameters
'stats'!\$H\$45	13	5	Interest tree volatility parameter
'cf'!\$H\$7	6	6	Bond cash flows

**Package: SRM**

[Interest rate process modelling](#)

The objective of the model is to analyze fixed income securities, specifically bonds. The template allows to set generic parameters for the interest rate tree but generally assumes specific inputs for the calculated bond prices and related interest rate options.

[Modelling bond prices](#)

The bond prices are modelled as follows:

1. **The bond terminal value.** The terminal value is generally assumed to be equal to the sum of the outstanding principal value and the accrued interest. The outstanding principal value depends on the (i) time period; (ii) applied amortization schedule; (iii) and exercised options. The accrued interest depends on the (i) time period; (ii) interest payment schedule; and (iii) outstanding balances used as a base for interest calculations.
2. **The bond coupon payments.** The bond coupon payments depend on time period and interest payment terms. Fixed interest payments are assumed (will be extended to variable interest rate).
3. **Redemption price.** The redemption price models the penalty that is applied in the case of bond option exercise. Generally, both call (early prepayment) and put (pay-on-demand) options involve a penalty that determines the bond redemption price.
4. **Amortization schedule.** The schedule determines the timing of bond principal repayments. Based on the amortization schedule, the outstanding principal balances are calculated.

**Package: SRM**

**Note: to resize the output arrays, press Ctrl+R**

[Markov tree objects and output](#)

```
IMarkovTreeR1 tree = Markov.tree(int T, double x0, Map states, Object Q);
```

Construct Markov tree object using the following paarmeters:

T - number of periods

x<sub>0</sub> - initial state,

states - Map: X-min - minimum state value, X-max - maximum state value; X-count - number of states; and X-type which is set either to *uniform* or *log-uniform* to describe how the states are generated within the range;

Q - object that describes transition probabilities (typically in the form of a function: (t, x) => {xi => pi} that mapps given period and state (t, x) into a mapping {xi => pi} between next period states and probabilities;

```
Object[][][] out = Markov.get(IMarkovTreeR1 tree, String key, Map<String, Object> params);
```

Return the value of a given field (referred to bya given key) of the Markov tree using the given set of parameters (if necessary). Keys include:

state-values - an array[][] that describes tree structure (contains a state value in a given cell if the cell belong to the tree or zero otherwise);

Q - returns an array of transition probabilities Q(x<sub>t</sub>, x<sub>t+1</sub>) for a given period t;

P - return an array of tree state probabilites