Valuation of CMO Securities

Introduction

A collateralized mortgage obligation (CMO) is a type of complex debt security that repackages and directs the payments of principal and interest from a collateral pool of mortgage loans to different types of securities. Legally, a CMO is a debt security issued by a special purpose entity (SPE) (thus it is not considered a debt owed by the institution creating and operating the SPE). The SPE is the legal owner of a set of mortgages, called a pool. Investors in a CMO buy bonds issued by the entity, and they receive payments from the income generated by the mortgages according to a defined set of rules. With regard to terminology, the mortgages themselves are termed collateral, 'classes' refers to groups of mortgages issued to borrowers of roughly similar credit worthiness, tranches are specified fractions or slices, metaphorically speaking, of a pool of mortgages and the income they produce that are combined into an individual security, while the structure is the set of rules that dictates how the income received from the collateral will be distributed. The legal entity, collateral, and structure are collectively referred to as the deal. Unlike traditional mortgage pass-through securities, CMOs feature different payment streams and risks, depending on investor preferences.

CMO securities offer the following advantages.

1. An average mortgage loan amount (say US$ 300,000) is quite material for an average investor. Issue of bonds collateralized by mortgage loans allows to set flexible principal amount on each bond. For example a US$ 300,000 mortgage loan can be split into 300 bonds with US 1,000 principal amount.
2. Mortgage loans are long-term (they typically have a 30-year maturity term). Investors typically prefer not to lock their funds on such a long period. CMO instruments are traded publicly and therefore allow investors to sell the bonds at any time.
3. Securities issued in a CMO deal typically have a flexible structure so that investors with different risk / return preferences can buy the securities.
4. Default risk s diversified in a pool of mortgage loans. Traditionally the CMO securities were viewed as having very good creditworthiness. However the market crash in 2008-2009 showed that defaults in a mortgage loan pool can be strongly correlated and collateral value of the houses can decrease significantly during the crisis. The credit rating on many CMO securities was downgraded from AA to CCC within a few month period during the market crash. Therefore potentially investment into CMO securities can be highly risky.

The major risks associated with the CMO securities are the following:

1. Default risk. House buyers default on their mortgages. After the default, the houses go into foreclosure and may often be sold at the price that is lower than the mortgage loan outstanding principal balances.
2. Prepayment (refinancing) risk. Mortgage loans are amortized over the 30-year maturity term. Moreover the loans can be prepaid without penalty partially or in whole at any time. The

mortgage loans are often prepaid when the mortgage market rates go down so that the mortgage can be refinanced at a lower rate. The risk is called the interest rate risk (or refinancing risk). Effectively the investment horizon in the CMO securities is less than 30-years. The prepayment risk must be reflected as the yield premium or price discount of the CMO security.

To address the above risks, the CMO deal structure specifies loss and principal allocation rules so that different securities within the CMO deal are protected differently against the default and prepayment risks.

CMO Deal Structure

Loss Allocation Rules

Support against default risk is performed using **credit tranching**, which is one of the forms of security credit protection. In the simplest case, credit tranching means that any credit losses will be absorbed by the most junior class of bondholders until the principal value of their investment reaches zero. If this occurs, the next class of bonds absorb credit losses, and so forth, until finally the senior bonds begin to experience losses. The subordinated (junior) securities are typically labelled with letter B, mezzanine securities are labelled with letter M, and senior securities are labelled with letter A. Additional credit protection can be added between senior securities and in some cases between different classes of securities.

In general the structure of credit protection is a combination of two rules:

1. **Hierarchical structure**, which specifies subordination between CMO securities with respect to allocation of recognized losses from the defaults on underlying mortgage loans;
2. **Pro-rata structure**, which assumes equal hierarchical order of the securities and allocates recognized losses to different securities proportionally to the securities outstanding balances.

In a typical CMO deal structure, the hierarchical loss allocation structure is specified by the junior - mezzanine - senior (B-M-A) subordination structure of securities and within the securities labelled with the same letter the losses are allocated on pro-rata basis.

Principal Allocation Rules

If a security is priced at premium on the issue date, then the coupon rate on the security exceeds the security yield rate equal to the sum of risk free rate and security risk premium. In the case of early prepayment investor may incur losses on the security since investor does not receive the coupon payments, which were expected when the security price premium was estimated. Therefore early prepayment may represent a risk to the investor who expects to receive the return to the investment in the form of coupon payments over an extended period of time.

The principal allocation rules are specified for the CMO deal so that to produce securities with different effective maturity term and amortization schedule. Investor who requires protection against early principal prepayment can purchase then the securities in the CMO deal that have an effective maturity term and amortization schedule that satisfies the investor's needs.

In general, the principal allocation rules can be quite complex (and typically are more complex than the loss allocation rules). Most principal allocation structures can be viewed as a combination of hierarchical structure, pro-rata structure, and **weight-based structure**. The hierarchical and pro-rata structures were described above. In the weight-based structure, the principal is allocated to different securities based on specific weights that do not depend on the securities outstanding balances. For example, 20% of all repaid principal amount in a given period is allocated to security A1 and the remaining 80% of repaid principal amount is allocated to other senior tranches on pro-rata basis.

Many CMO deals specify a "**crossover**" date or "**senior credit support depletion**" date, which is defined as the date when the balances of all junior and mezzanine securities are reduced to zero. The principal allocation rule for these CMO deals typically switches from original complex rules to a simple pro-rata allocation rule, which applies to all remaining senior securities in the deal. We call the pro-rata rule a "simple" rule since it allows to analyze each senior security independently from all other securities in the deal. The principal prepayment rate, which was estimated for the whole deal based on total deal securities balances and total prepaid balances in each period, applies to each individual security if the prepaid principal is allocated on pro-rata basis.

Modelling Losses and Principal Allocation Rules

Currently the loss allocation and principal allocation rules are modelled in Alexa's CMO valuation application independently from each other. This generally imposes a strong restriction on modelled structures CMO as, for example, it does not allow to model the CMO deals with senior credit support depletion date in which the principal allocation rules depend on the total losses and respectively total balances of junior and mezzanine balances. The valuation for these CMO deals can be performed only after the senior credit support depletion date, when the principal allocation rule switched to the pro-rata rule.

Alexa's CMO valuation application is also generally not applicable to the CMO deals with multiple security classes, in which the loss and principal allocation in some security classes depend on the allocation in other security classes. The Alexa's CMO valuation application assumes a single loss and principal repayment rate that is applicable to the modelled CMO deal, while multiple security classes with cross-class allocation rules dependence require to model the loss and principal repayment rate for each security class individually.

The loss (principal) allocation rule in Alexa's CMO valuation application is modelled as a combination of hierarchical, pro-rata, and weight-based structures. The securities in the CMO deal are grouped together depending on which of the three structures is applied to each specific group. The structure is generally modelled as a tree of security groups. Each group can contain either a security or another group of securities. The allocation rule within each group can be different and is specified by one of the above three rules.

To illustrate the modelling approach, suppose that we need to model a simple loss allocation structure in which the losses are allocated based on the subordination structure of the securities. The total group of securities, labelled as G, is divided into three groups: group B of junior securities, group M of mezzanine securities, and group A of senior securities. Within each group B, M, and A the losses are allocated on pro-rata basis. Within group G the losses are allocated using hierarchical structure: first to B, then to M, and finally to A.

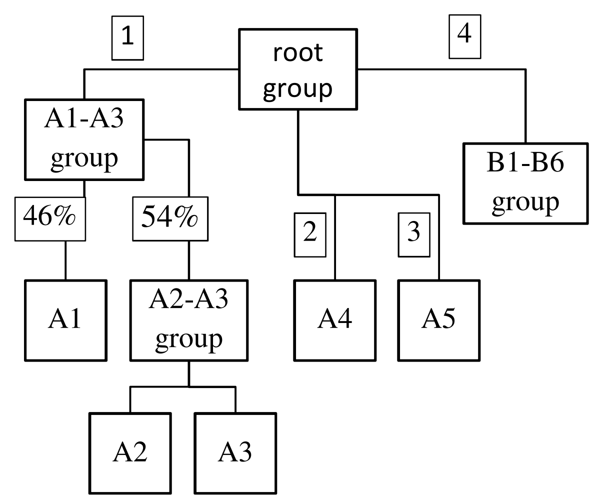
Describing Losses and Principal Allocation Rules

The allocation rules are described in Alexa's CMO valuation application using a table that lists all the groups and securities in the CMO deal, the group tree structure, and the allocation rule within each group. An illustration of the table and related group tree are shown in the exhibit below.

Table representation of the loss allocation rules

Tranche Name	Outstanding Balance	Loss Parent	Loss Rank	Loss Weight
B1-B6	20,000,000	root	4	
A1	50,000,000	A1-A3		46.0%
A2	20,000,000	A2-A3		
A3	60,000,000	A2-A3		
A4	100,000,000	root	3	
A5	20,000,000	root	1	
A1-A3	130,000,000	root	2	
A2-A3	80,000,000	A1-A3		54.0%

Group-tree representation of the loss allocation rules



The table lists all securities and security groups in the first column (the column includes the list of all groups used to describe the allocation structure. In the example, there are three groups used to describe the structure: B1-B6, A1-A3, and A2-A3). The second column specifies respectively each security or security group balance. The third column specifies the parent group of each respective security or security group. Root is the name for the ultimate parent group that contains all securities in the deal. The fourth column describes the hierarchical ranking within the group and the last column describes the weight-based structure within the group. If a specific security or security group in the first column is assigned neither the rank in column four nor the weight in column five, then it is assumed that pro-rata structure is applied to the security (or security group).

The structure can be applied to describe generic loss and principal allocation structures. The application also performs validation of the structure. After the simulation of loss and principal cash flow allocation to each security and each security group, the application performs the following two validation tests:

1. The application tests that each parent group is balanced in each period: the parent group balance equals to the sum of its child group balances;

2. The application tests that allocation across the children of each parent group is consistent with the allocation rule assigned to the parent group.

Hedging CMO Securities with CDS Contracts

Overview

The credit default swap provides an additional credit support structure for a given CMO security. The support is structured as follows. The senior security ("A") is divided into three tranches: junior ("A.J"), mezzanine ("A.M" or "A.CDS"), and senior ("A.S"). The balance of the A.J tranche is typically selected to match the price discount at which security A was acquired. From accounting perspective, the losses in the A.J tranche are covered by the security A price discount (since the total value of the security is recorded in the balance sheet at the purchase price and not at the nominal value). The losses in the A.CDS tranche are covered by the CDS contract. The purpose of the A.J and A.CDS tranches is to provide additional credit support for the A.S tranche. All losses in A securities are first accumulated in A.J and then in A.CDS tranches. The sizes of the A.J and A.M sub-tranches are determined using "*attachment point*" and "*detachment point*". For example, if attachment point equals 5% and detachment point equals 15%, then the size of the A.J tranche is equal to 5% of the A security size and size of the A.M tranche is equal to 10% (10% = 15% - 5%) of the A security.

The order of principal payments is specified by the master agreement that provides the terms of the CDS instruments. A standard assumption is that the tranche A.S is repaid first, then, after the tranche A.S balance is reduced to zero, tranche A.CDS is repaid. Tranche A.J is repaid last.

The price of the CDS instrument depends on the distribution of losses in the A.CDS sub-tranche of the A security. The key parameters that affect the distribution of losses in the A.M tranche are the following.

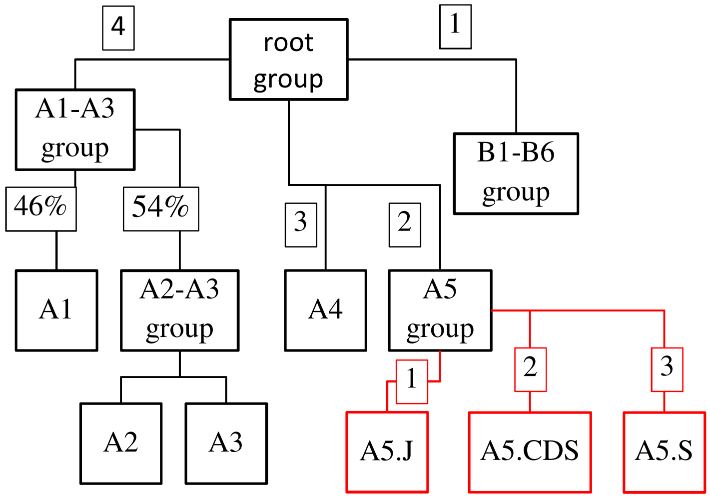
1. The outstanding balances and attachment and detachment points of security A;
2. Loss and principal allocation structure of the CMO deal (which determines the loss and principal allocation to the A security);
3. Parameters of loss and principal repayment distributions estimated for the total pool of underlying mortgage loans;

Each CDS contract can be equivalently interpreted as an **insurance policy** underwritten on the CMO security. The attachment point is interpreted as the insurance deductible and the detachment point is interpreted as the insurance maximum limit.

Alternatively, the CDS contract can be interpreted as a portfolio of long call option with strike equal to attachment point and short call option with a strike equal to detachment point, which is underwritten for the CMO security (the portfolio is also known as a **bear spread**).

CDS Contract Valuation

CDS contract can be modelled as follows. First the tranching of the security is incorporated as a part of the CMO deal structure. For example, suppose that security A5 in the example above was tranched into junior, CDS and senior sub-loans. Then the modified CMO deal structure would be represented using the following tree.



The modelled allocation of losses and principal and interest payments includes now securities (tranches) A5.J, A5.CDS, and A5.S. Note that by definition

$$NPV\ L + NPV\ I^{rf} + NPV\ P = Par$$

(The net present value of the security losses L, principal payments P and risk free interest payments I^{rf} equal to par value, which is by definition equal to the security nominal amount and accrued interest). The value of the security V is equal to

$$NPV\ C + \ NPV\ P = V$$

where C is security coupon payments. Therefore,

$$NPV\ L - NPV\ \left(C - I^{rf}\right) = Par - V$$

where $C - I^{rf}$ can be interpreted as the risk spread paid on the security. Based on the above equation, if the CDS contract compensates all losses in the CDS tranche and receives the risk spread on the CDS tranche as a periodic payment, then the fixed price of the CDS contract is equal to the difference in par value and the contract price estimated using NPV valuation approach (this is the equation used by JP Morgan Chase for CDS contract valuation).

In practice the periodic payment on a CDS contract C^{CDS} may be equal to zero, risk spread, coupon rate, or other selected value. A general equation for the CDS price based on NPV valuation is

$$V^{CDS} = NPV\ L - NPV\ C^{CDS}$$

CMO Cash Flow Model

Overview

The cash flows are estimated using the following sequence of calculations in the CMO valuation application (the cash fluctuations in the application are consistent with the cash flow data analyzed for different actual CMO deals).

1. Set beginning of period $t+1$ security balances equal to the end of period t security balances;
2. Calculate the coupon payments based on the beginning of period security balances;
3. Calculate losses in the balances assuming constant loss rate (estimated based on historical loss data). Adjust balances for the default losses;
4. Calculate mandatory amortization for the outstanding balances. Adjust balances for mandatory amortization;
5. Calculate voluntary prepayment of the outstanding balances. Adjust balances for voluntary prepayment. The adjusted balances are end of period $t+1$ balances.

Cash flow calculations

The cash flows in the CMO securities are calculated by following the steps below.

- Coupon payments, losses in security balances, and voluntary prepayment in security balances are calculated directly as

$$C_t = c_t \times B_t$$
$$L_t = \alpha \times B_t$$
$$X_t^v = \gamma \times B_t$$

  where c_t is coupon rate in period t, and α and γ are fixed loss and voluntary prepayment rates.

- For floating coupon payments calculated as Libor + spread, estimated forward Libor rates based on floating-to-fixed swap curve.[1]
- Mandatory amortization amount X_t^m is estimated as follows.

$$X_t^m = \frac{c \times B_t \times (1+c)^N}{(1+c)^N - 1} - c \times B_0 = \frac{c \times B_0}{(1+c)^N - 1}$$

  (The mandatory amortization is estimated based on the following principle: the payment in each period is fixed and equal to $X = c \times B_t + X_t^m$. The next period balance (assuming to zero losses and voluntary prepayment) is equal to $B_{t+1} = B_t - P_t = B_t \times (1+c) - X$. The last period balance is equal to $B_{t+n} = B_t \times (1+c)^n - X \times (1+c)^{n-1} - \cdots - X = B_t \times (1+c)^n - X \times \frac{(1+c)^n - 1}{c} = 0$. Therefore the fixed payment is equal to $X = \frac{c \times B_t \times (1+c)^N}{(1+c)^N - 1}$ and planned amortization is equal to the difference in fixed payment and coupon payment).

  The end of period outstanding balances in each security are estimated as follows

$$B_t = B_{t-1} - L_t - X_t^m - X_t^v$$

- The cash flows are allocated starting from the CMO deal root security group into each child security group and then sequentially from each parent group into each child group, based on the loss and principal repayment allocation rules set in each parent group.

[1] Note that forward Libor rates are different from projected expected Libor rates. Applying forward rates is based on the assumption that investor into the floating-rate security hedges the volatility in Libor rates by entering in Libor forward contracts.

Parameter Estimation

Loss and principal prepayment rates are estimated using total balances, total losses, total repaid principal, and weighted-average effective coupon rate in the CMO deal. (The weighted-average effective coupon rate is applied to estimate the mandatory total principal amortization amount).

Loss Rate Estimation

Distribution of Losses

Distribution of loss rates is derived based on the following simple model.

Suppose that there are n mortgages in total and the default on each mortgage is independent across mortgages and across periods. All mortgages are assumed to be equal in size (the size of the average mortgage is denoted as M). In the event of default, fraction $R < 1$ of the mortgage value is recovered from the sale of the house.[2] The loss on a specific mortgage, L_{it}, is a random variable distributed as

$$L_{it} = \begin{cases} (1-R)M, & p \\ 0, & 1-p \end{cases}$$

where p is the probability to default in a given period on a specific mortgage. For a fixed probability p and large enough number of mortgages the total losses converge to the Normal distribution. However, in practice the probability of default p is small. Therefore, we assume that as n increases, the factor $\alpha = p \times n$ stays constant so that the distribution of total loss converges to the Poisson distribution with parameter α. The Poisson distribution is described by the following formula.

$$P_k = e^{-\alpha}\frac{\alpha^k}{k!}$$

In addition, if R is interpreted as the probability of recovery conditional on default, then the distribution L_{it} can be equivalently presented as

$$L_{it} = \begin{cases} M, & p(1-R) \\ 0, & 1-p(1-R) \end{cases}$$

so that the total losses are distributes as Poisson distribution with recovery-adjusted frequency parameter $\tilde{\alpha} = p(1-R) \times n$. The advantage of the loss rate representation with the recovery-adjusted frequency parameter $\tilde{\alpha}$ of Poisson distribution is that it does not require to estimate parameter R but instead blends parameter R into the parameter $\tilde{\alpha}$ of the Poisson distribution. The total losses in period t are estimated as

$$L_t = \sum_i L_{it} \sim M \times P(\tilde{\alpha})$$

where M is the average size of the outstanding mortgage balances and $B = nM$ represents outstanding total mortgage balances. In the simulations, parameter M is estimated at the issue date as the average mortgage balance and then in each consecutive period the parameter is adjusted so that the ratio $\frac{M_t}{B_t}$ is constant at each period of time (the loss amount is proportional to the current outstanding balances).

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[22] Note that if the value of the house exceeds the outstanding nominal amount of the mortgage loan, the mortgage holder can either sell or refinance the house and repay the loan. Therefore the borrower will default on the loan only if the value of the house will not cover the remaining loan balances (if $R < 1$).

Estimation of Loss Parameter

There are two alternative approaches to estimate the loss rate parameter:

1. Direct approach based on Poisson distribution and historical sample of losses in the CMO deal. The ratio of loss to average loan balance, $\frac{L_t}{\frac{1}{n}B_t}$ is estimated for each period. The Poisson distribution with recovery-adjusted Poisson parameter $\tilde{\alpha}$ is estimated based on the constructed sample of loss rates;
2. Indirect approach based on security public market prices. Security are traded in the market. The prices of the securities are either directly reported or estimated for example by Bloomberg. Loss parameter generally has a one-sided impact on the security prices: the higher the loss rate, the lower is the price of each security. Therefore there is a single loss rate that minimizes the sum of squared between the actual observed security prices and the prices estimated using NPV valuation approach based on the cash flow model.

Principal Prepayment Rate Estimation

The principal payments consist of two separate parts:

1. Mandatory principal amortization. Each period a mortgage borrower pays a fixed amount so that the mortgage loan principal is repaid completely on the maturity date. The condition determines the fixed payment made by the borrower in each period, which includes the coupon payment for the period and amortized principal amount. The mandatory principal amortization is calculate as the difference between the estimated fixed monthly payment and the coupon payment.
2. In addition to mandatory principal amortization, the mortgage loan borrower has an option to prepay in part or the whole mortgage loan. The voluntary principal prepayment depends in general on multiple economic and borrower-specific factors and is modelled as a random process. The prepayment rate determines how quickly the outstanding balances are reduced in each security and, therefore, represent a significant factor in the valuation of the security risks.

Note that in Alexa's tool the balances are adjusted first for the losses and then for the mandatory principal amortization prior to principal prepayment rate estimation.

The prepayment rate generally does not have a one-sided impact on the security prices. A higher prepayment rate may have both a positive or negative effect on the security prices and the effect may be different for different securities. The impact depends on the CMO deal structure and on the estimated loss rate. With high loss rate, high prepayment rate may result in lower losses and as a result in higher security price. With low loss rate, high prepayment rate may result in lower effective maturity term and as a result in lower security price.

Because of the uncertain impact of the prepayment rate on security prices, the recommended approach is to estimate the prepayment rate parameter based on the historical sample of prepayment rates in the CMO deal.

PSA Prepayment model

The PSA Prepayment Model is a prepayment scale developed by the Public Securities Association in 1985 for analyzing American mortgage-backed securities. The PSA model assumes increasing prepayment rates for the first 30 months after mortgage origination and a constant prepayment rate thereafter. This approximates real-world experience that during the first few years, mortgage borrowers:

- are less likely to relocate to a different home,
- are less likely to refinance into a new mortgage, and
- are less likely to make extra payments of principal.

The standard model (also called "100% PSA") works as follows: Starting with an annualized prepayment rate of 0.2% in month 1, the rate increases by 0.2% each month, until it reaches 6% in month 30. From the 30th month onward, the model assumes an annualized prepayment rate of 6% of the remaining balance. Each monthly prepayment is assumed to represent full payoff of individual loans, rather than a partial prepayment that leaves a loan with a reduced principal balance.

Variations of the model are expressed in percent; e.g., "150% PSA" means a monthly increase of 0.3% in the annualized prepayment rate, until the peak of 9% is reached after 30 months. The months thereafter have a constant annualized prepayment rate of 9%.

Summary of CMO Valuation Model

CMO securities valuation is performed by following the steps below.

1. Based on CMO prospectus, estimate the security structure:
   i. Summarize the list of securities in the CMO deal;
   ii. Summarize the outstanding balances of the securities;
   iii. Summarize the coupon payment on the securities;
   iv. Summarize loss allocation rules and construct the respective security group tree and related application input table;
   v. Summarize principal allocation rules and construct the respective security group tree and related application input table;
2. Download security-specific data on losses, principal payments, outstanding balances, and coupon payments. Validate consistency of the historical data against the allocation rules summarized above;
3. Download available market prices for the traded securities in the CMO deal;
4. Estimate mandatory principal amortization for the total balances of the CMO deal;
5. Estimate the historical loss rate based on either (i) historical loss rate data or on (ii) security market prices;
6. Estimate the historical voluntary principal prepayment rate based on historical voluntary prepayment rate data;
7. Estimate the cash flows in each security of the CMO deal based on the estimated parameters and formulas described above;
8. Estimate the risk-free discount rates applicable to the CMO securities.
   i. Estimate risk-free yield term structure (based for example on Treasury rates);
   ii. Convert the yield curves into the related risk-free discount factors D_t^*.
9. **Deterministic approach**. Estimate the price of each security as the NPV of the related security cash flows. For the CDS contracts that hedge the CMO securities, estimate the NPV of losses in CDS-covered tranches and NPV of the fixed periodic payments to the CDS seller. Estimate the CDS price as the difference between the two NPV values;

10. **Monte-Carlo simulation approach.**
    i. Construct samples of loss rate and principal prepayment rate based on the estimated distributions of the loss rate and principal prepayment rate. If the loss rate is estimated based on the market prices of the securities, then construct the sample of principal prepayment rates only;
    ii. Construct the sample of security prices (CDS prices) for each given loss rate and principal prepayment rate as described in step 9.
    iii. Estimate the price of each security in the CMO deal as the average of the sample security prices.

Note that the Monte-Carlo approach takes into account not only the average loss and principal prepayment rate in the security valuation, but also variation in the rates. Therefore the Monte-Carlo approach is generally more accurate and more robust for security price valuation purposes.

Example

For illustrative purposes, we show how the valuation is performed for the BOAMS 2007-2 CMO deal (Banc of America Mortgage Securities, Inc. Mortgage Pass-Through Certificates, Series 2007-2). The CMO deal is described as follows.

- Prospectus link and the link to Excel working file with the CMO deal historical cash flows.
- The deal was issued in April 2007 and original total nominal principal amount mortgage pool loans was US$416,134,218. As of Sep 2017, the remaining balance of the CMO deal was US$72,295,042. The balances of junior B1-B3 and mezzanine M1-M2 securities were exhausted in August 2011;
- CMO deal security structure. The deal has one class of senior securities A1 to A14. The security balances are summarized in the exhibit below.

Security class	Nominal balance amount				Allocation shares		Coupon rate
	Issue date		September 2017		principal	Loss	
	in US$	in %	in US$	in %	In %	In %	In %
Classes B1 to B3	6,732,000	1.62%	0				
Classes M1 to M2	9,678,000	2.33%	0				
Class A1	40,001,000	9.61%	7,266,267	10.05%	10.10%	10.15%	1.24%
Class A3	37,306,000	8.96%	10,940,373	15.13%	15.21%	15.29%	6.13%
Class A4	480,000	0.12%	0				
Class A5	69,246,000	16.64%	565,527	0.78%	0.79%	0.79%	5.62%
Class A6	20,000,000	4.81%	2,722,129	3.77%	3.78%	3.80%	5.87%
Class A7	49,528,000	11.90%	17,972,954	24.86%	24.99%	25.12%	5.62%
Class A9	25,656,000	6.17%	0				
Class A10	9,812,500	2.36%	2,848,599	3.94%	3.96%	3.98%	5.97%
Class A11	9,812,500	2.36%	2,877,618	3.98%	4.00%	4.02%	6.28%
Class A12	6,413,000	1.54%	1,861,714	2.58%	2.59%	2.60%	6.13%
Class A13	122,484,000	29.43%	24,204,957	33.48%	33.65%	33.82%	6.13%
Class A14	1,550,000	0.37%	0				
Class 30-PO	7,435,148	1.79%	1,034,904	1.43%			
Total	**416,134,148**	**100%**	**72,295,042**	**100%**	**100%**	**100%**	

A2 and A8 are interest only (IO) securities. A9 security is composed of three tranches: A-9-1, A-9-2 and A-9-3. A1 is a floating rate security. Classes A3 and A13 have 6.0% fixed coupon rate, Classes A5 and A7 have 5.5% fixed coupon rate, class A6 has 5.75% fixed coupon rate, and classes A10-A12 have on average 6.0% fixed coupon rate. (The estimated effective coupon rate, presented in the last column, is higher by approximately 13bps for each class).

The CMO deal has a credit support depletion date after which the complex initial principal allocation rules switch to simple pro-rata allocation rules. The credit support depletion date became effective on 25 August 2011. The pro-rata allocation rules are validated in the above exhibit. The percentage of allocated total losses and principal balances to each security approximately equals to the percentage of the security outstanding balances relative to the deal total outstanding balances. The allocation of losses and principal amount in one month prior and one month after credit support depletion date are presented in the exhibit below.

Certificate class	July 2011				September 2011			
	Nominal balance		principal	loss	Nominal balance		principal	loss
	in US$	in %	In %	In %	in US$	in %	In %	In %
Classes B1 to B3								
Classes M1 to M2	752,053	0.30%	0.00%	99.97%				
Class A1	25,029,936	9.98%	10.17%	0.00%	24,233,024	10.01%	9.79%	10.17%
Class A3	37,306,000	14.87%	0.00%	0.00%	36,300,621	15.00%	14.59%	0.00%
Class A4	480,000	0.19%	0.00%	0.00%	234,363	0.10%	0.19%	19.34%
Class A5	1,655,847	0.66%	66.39%	0.00%	1,611,223	0.67%	0.65%	0.00%
Class A6	8,907,853	3.55%	10.89%	0.00%	8,667,790	3.58%	3.48%	0.00%
Class A7	49,528,000	19.74%	0.00%	0.00%	48,193,244	19.91%	19.38%	0.00%
Class A9	13,144,210	5.24%	12.29%	0.00%	12,431,941	5.14%	5.14%	29.75%
Class A10	9,812,500	3.91%	0.00%	0.00%	9,500,086	3.93%	3.84%	3.99%
Class A11	9,812,500	3.91%	0.00%	0.00%	9,548,058	3.94%	3.84%	0.00%
Class A12	6,413,000	2.56%	0.00%	0.00%	6,208,821	2.57%	2.51%	2.61%
Class A13	83,378,232	33.23%	0.00%	0.00%	80,723,606	33.35%	32.62%	33.88%
Class A14	0							
Class 30-PO	4,675,237	1.86%			4,385,340	1.81%		
Total	**250,895,368**	**100%**	**100%**	**100%**	**242,038,117**	**100%**	**100%**	**100%**

In addition to credit support from junior and mezzanine tranches, there is credit support between senior tranches. Super senior certificates A3 and A11 are supported by class A4 and super senior certificates A5, A6 and A7 are supported respectively by classes A9-1, A9-2 and A9-3.

- Loss allocation structure. As showed in the exhibits above, the losses are allocated on pro-rata basis to securities A1, A10, A12 and A13 after the credit support depletion date. The losses are allocated on pro-rata basis to all securities after November 2014 when the balances of all super senior support securities (which include A4 and A9 classes) are reduced to zero;
- Principal allocation structure. As showed in the exhibit above, the principal payments are allocated on pro-rata basis to all securities in the deal after the credit support depletion date. Prior to the credit support depletion date the principal allocation rules are more complex with the largest share of repaid principal amount being allocated to class A5;

- Principal prepayment rate parameter.
- Loss rate parameter.
- Cash flow model;
- NPV valuation:

Appendix

Bloomberg functions

List of Bloomberg functions used to retrieve CMO securities data is provided below.

- CPD:
- DPD:
- LLD:
- PRO: CMO deal prospectus;
- SEV:
- HP (GP): historical prices as a table (or as a graph).

An Approximate Approach to CDS Pricing

In this section, we describe a high-level approach that does not require modelling the credit enhancement / principal prepayment structure of MBS loan tranches and does not require running Monte-Carlo simulations to assess the value of the CDS instruments. The valuation approach is based on the assumption that the expectation of the losses in A tranche can be estimated from the Bloomberg reported price of the tranche and that the standard deviation of the losses is typically relatively small and not sensitive to modelling parameters. Basically, the resulting distribution of losses that determine the prices of the A4.J, A4.M, and A4.S sub-tranches is relatively insensitive to the modelling approach and modelling parameters given that the price of the A tranche is fixed.

Below we provide a simplified model of CDS pricing. Suppose that the A tranche is modelled as amortized loan where the loss L and the monthly amortized amount X are fixed and constant each month. Losses start to be recognized starting from date T^L. Similarly, the principal amortization starts from period T^A. The values of T^L and T^A for each tranche depend on credit support and principal amortization support balances as well as on the loss and amortization rates. By T^{min} and T^{max} we denote $T^{min} = min(T^L, T^A)$ and $T^{max} = max(T^L, T^A)$.

Suppose that i represents the coupon rate and y represents the risk-free discount rate. If original balance is B_0 then outstanding balance in period $t \geq T^{max}$ is $B_t = B_{T^{max}} - (t - T^{max})(L + X)$. The balance is reduced to zero ($B_T = 0$) in period $T = T^{max} + \frac{B_{T^{max}}}{L+X}$. The NPV value[3] of the amortized principle amounts is

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[3] The simplest generic approach to estimate different discounted sums expressions is via the function $\varphi(x) = \sum_{t=1,\dots,T} \frac{x^t}{(1+y)^t} = \frac{x}{1+y-x}\left(1 - \left(\frac{x}{1+y}\right)^T\right)$. Using the function formula, we can estimate that $\sum_{t=1,\dots,T} \frac{1}{(1+y)^t} = \varphi(1) = \frac{1}{y}\left(1 - \left(\frac{1}{1+y}\right)^T\right)$, and $\sum_{t=1,\dots,T} \frac{t}{(1+y)^t} = \varphi'(1) = \frac{1+y-x-x(-1)}{(1+y-x)^2} \times \left(1 - \left(\frac{x}{1+y}\right)^T\right) - T\frac{x}{1+y-x} \times \frac{1}{1+y} \times \left(\frac{x}{1+y}\right)^{T-1} = \frac{1+y}{y^2}\left(1 - \left(\frac{1}{1+y}\right)^T\right) - T\frac{1}{y}\left(\frac{1}{1+y}\right)^T$

$$P^X = \sum_t \frac{X}{(1+y)^t} = \frac{1}{(1+y)^{T^{max}}}\left[\frac{X}{y}\left(1 - \frac{1}{(1+y)^{T-T^{max}}}\right)\right].$$

Coupon payment in period t is $I_t = iB_t = i\big(B_0 - t(L+X)\big)$. The NPV of the coupon payments is

$$P^I = \frac{iB_0}{y}\left(1 - \frac{1}{(1+y)^T}\right) - \frac{i(L+X)}{y}\left(\frac{1+y}{y}\left(1 - \left(\frac{1}{1+y}\right)^T\right) - T\left(\frac{1}{1+y}\right)^T\right)$$

The price of the amortized loan is estimated as

$$P = \frac{X + iB_0}{y}\left(1 - \frac{1}{(1+y)^T}\right) - \frac{i(L+X)}{y}\left(\frac{1+y}{y}\left(1 - \left(\frac{1}{1+y}\right)^T\right) - T\left(\frac{1}{1+y}\right)^T\right)$$

Together with the equation

$$T = \frac{B_0}{L+X}$$

we can estimate the values of X and L and derive the expected losses in the A tranche and, after making certain assumptions on the standard deviation of the losses, we can derive the price of the sub-tranches of the A tranche. Substituting out the $L+X$ in the second equation, we obtain the following equation for the amortized amount X:

$$P = \frac{X + iB_0}{y}\left(1 - \frac{1}{(1+y)^T}\right) - \frac{\frac{iB_0}{T}}{y}\left(\frac{1+y}{y}\left(1 - \left(\frac{1}{1+y}\right)^T\right) - T\left(\frac{1}{1+y}\right)^T\right)$$

or

$$X = Py + \frac{iB_0}{T} \times \left(\frac{1+y}{y}\left(1 - \left(\frac{1}{1+y}\right)^T\right) - T\left(\frac{1}{1+y}\right)^T\right) - iB_0 \times \left(1 - \frac{1}{(1+y)^T}\right)$$

After simplifying we obtain

$$X = Py + \frac{iB_0}{T}\left(\frac{1+y}{y}\left(1 - \left(\frac{1}{1+y}\right)^T\right) - T\right) \tag{1.1}$$

and

$$L = \frac{B_0}{T} - X \tag{1.2}$$

In the above equations, the number of periods T is measured in months, the coupon rate i and risk free discount rate y are monthly rates.