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Swaps

Overview[1]

Description

Definition: A swap is a derivative contract where two parties exchange financial instruments. Most swaps are derivatives in which two counterparties exchange cash flows of one party's financial instrument for those of the other party's financial instrument. Specifically, two counterparties agree to exchange one stream of cash flows against another stream. These streams are called the legs of the swap. The swap agreement defines the dates when the cash flows are to be paid and the way they are accrued and calculated. Usually at the time when the contract is initiated, at least one of these series of cash flows is determined by an uncertain variable such as a floating interest rate, foreign exchange rate, equity price, or commodity price.

The cash flows are calculated over a notional principal amount. Contrary to a future, a forward or an option, the notional amount is usually not exchanged between counterparties. Consequently, swaps can be in cash or collateral. Swaps are among the most heavily traded financial contracts in the world: the total amount of interest rates and currency swaps outstanding is more than $348 trillion in 2010, according to Bank for International Settlements (BIS).

Most swaps are traded over-the-counter (OTC), "tailor-made" for the counterparties. Some types of swaps are also exchanged on futures markets such as the Chicago Mercantile Exchange, the largest U.S. futures market, the Chicago Board Options Exchange, Intercontinental Exchange and Frankfurt-based Eurex AG.

The five generic types of swaps, in order of their quantitative importance, are: interest rate swaps, currency swaps, credit swaps, commodity swaps and equity swaps. There are also many other types of swaps.

- **Interest rate swaps**.
- **Currency swaps**. A currency swap involves exchanging principal and fixed rate interest payments on a loan in one currency for principal and fixed rate interest payments on an equal loan in another currency;
- **Commodity swap**. A commodity swap is an agreement whereby a floating (or market or spot) price is exchanged for a fixed price over a specified period. The vast majority of commodity swaps involve crude oil;
- **Total return swap**. A total return swap is a swap in which party A pays the total return of an asset, and party B makes periodic interest payments. The total return is the capital gain or loss, plus any interest or dividend payments. Note that if the total return is negative, then party A receives this amount from party B. The parties have exposure to the return of the underlying stock or index, without having to hold the underlying assets. The profit or loss of party B is the same for him as actually owning the underlying asset;

[1] https://en.wikipedia.org/wiki/Swap_(finance)

- **Swaption**. An option on a swap is called a swaption. These provide one party with the right but not the obligation at a future time to enter into a swap;

Interest Rate Parity

Interest rate swaps are effectively derived from the interest rate parity:

$$i_t^F \approx i_t^D + f_t$$

where

$$f_t = \frac{F_t}{S} - 1$$

is the forward rate. The interest rate period can be viewed as a one-period approximation of an interest rate swap. The interest rate parity is based on the following arbitrage argument. Suppose that two debt transactions are traded in the market that have all term equal with exception of the debt currency. Both debt transactions mature in the next period. We also assume that debt is risk free. Then investor has two alternative strategies:

- **Strategy A**: invest in the debt traded in domestic currency. The return from the investment strategy is $1 + i_t^D \times \Delta_t$, where Δ_t is the day count;
- **Strategy B**: invest in the debt traded in domestic currency and hedge all currency risk. The return from the investment strategy is $(1 + i_t^F \times \Delta_t) \times \frac{S}{F_t}$

The two strategies must produce the same return. The condition is described by the following equation.

$$1 + i_t^F \times \Delta_t = (1 + i_t^D \times \Delta_t) \times (1 + f_t \times dt) \approx 1 + i_t^D \times \Delta_t + f_t \times dt$$

In practice, the maturity of the cross-currency swap exceeds one period. The simple equation above must be replaced then by a multi-period cash flow model and the swap rate is derived from the cash flow model. Conceptually however the cash flow model implements the same two investment strategies: (i) invest in debt denominated in domestic currency and (ii) invest in comparable debt denominated in foreign currency and hedge all FX risk with FX forward contracts. The approach is described in more detail in the notes below.

Valuation

The value of a swap is the net present value (NPV) of all estimated future cash flows. A swap is worth zero when it is first initiated, however after this time its value may become positive or negative. There are two ways to value swaps: in terms of bond prices, or as a portfolio of forward contracts.

NPV valuation approach is a direct approach to estimate the swap FMV. Since NPV valuation involves future cash flows, it requires using forward contracts that value future cash flows. NPV valuation requires following inputs:

- Discount factors;
- Forward prices used to value future cash flows;
- Terms of the swap contract.

The valuation approach based on bond prices is an indirect valuation approach that uses bond price information to infer underlying forward prices and then perform arbitrary swap contracts. Alternatively, standard swaps can be replicated directly using portfolios of bonds selected in a such way so that to replicate the swap cash flows.

Fixed-to-Fixed Cross-Currency Swaps

In a fixed-to-fixed cross-currency swap, coupon and principal payments of a bond in currency A into coupon and principal payments of a bond denominated in currency B. To estimate the payments denominated in another currency, cross-currency (FX) forward rates must be obtained or estimated. In the analysis below we assume that coupon payments are made in both legs of the cross-currency swap using annual frequency and ACT/ACT day count basis. The maturity term T is assumed to be a whole number of years.

After the cross-currency swap premium/discount is estimated for each $T = 1, 2, \ldots,$ number of years, the swap premium/discount for a fractional maturity term t is estimated by interpolating the swap premium/discounts with integer number of years $\lfloor t \rfloor$ and $\lceil t \rceil = \lfloor t \rfloor + 1$. The swap premium/discounts are estimated sequentially for each maturity term $T = 1, 2, \ldots,$ as described below.

NPV Approach

Formulas

Suppose that $F_t^{A,B}$ is the FX forward rate from currency A to currency B and $S^{A,B}$ is FX spot rate. Suppose also that c^A are coupon payments denominated in currency A and principal amount is normalized to one. The principal can be converted into currency B, $X^B = 1 \times S^{A,B}$, and coupon and principal payments can be converted then back into currency A using forward rates. The NPV of the two cash flows must be equal to each other.

In the case of a single period, the equation is described as follows.

$$D_T \times (1 + c^A \times \Delta_T) = D_T \times (1 + c^B \times \Delta_T) \frac{S^{A,B}}{F_T^{A,B}}$$

or

$$c^B = \frac{1}{\Delta_T} \times \left(\frac{F_T^{A,B}}{S^{A,B}} - 1 \right) + c^A \times \frac{F_T^{A,B}}{S^{A,B}} \qquad \text{(swp.1)}$$

Equation (swp.1) is applied for maturities $T < 1$.

In general case, the equation is described as follows.

$$\sum_{t=1,\ldots,T-1} D_t \times c^A \times \Delta_t + D_T \times (1 + c^A \times \Delta_T)$$
$$= \sum_{t=1,\ldots,T-1} D_t \times \frac{c^B \times \Delta_t \times S^{A,B}}{F_t^{A,B}} + D_T \times (1 + c^B \times \Delta_T) \frac{S^{A,B}}{F_T^{A,B}}$$

After rearranging the terms, the swapped coupon rate c^B denominated in currency B can be estimated using the following linear transformation of coupon payment c^A denominated in currency A.

$$c^B = \frac{1}{\left[\sum_{t=1,\dots,T} D_t \times \frac{S^{A,B}}{F_t^{A,B}} \times \Delta_t\right]} \times \left(D_T \times \left(1 - \frac{S^{A,B}}{F_T^{A,B}}\right) + c^A \times \sum_{t=1,\dots,T} D_t \times \Delta_t\right) = a_{0,T} + c^A \times a_{1,T} \quad \text{(swp.2)}$$

where

$$a_{0,T} = \frac{1}{\left[\sum_{t=1,\dots,T} D_t \times \frac{S^{A,B}}{F_t^{A,B}} \times \Delta_t\right]} \times D_T \times \left(1 - \frac{S^{A,B}}{F_T^{A,B}}\right) \quad \text{(swp.3)}$$

is an intercept and

$$a_{1,T} = \frac{1}{\left[\sum_{t=1,\dots,T} D_t \times \frac{S^{A,B}}{F_t^{A,B}} \times \Delta_t\right]} \times \sum_{t=1,\dots,T} D_t \times \Delta_t \quad \text{(swp.4)}$$

The linear relationship is estimated sequentially each maturity term $T \geq 1$.

Equations (swp.1) through (swp.4) describe a linear relationship between the interest rates denominated in different currencies. The linear relationship is estimated for each date and each maturity term based on respective forward and discount rates and applied then to calculate the swap for each interest rate c^A.

Cash Flows

The derived above equations present the relationship between bond coupons rates denominated in different currencies. The swap calculations can also be illustrated using the cash flows estimated for both legs of the swap instrument. The cash flows are presented by the following exhibit.

Period	Discounts	Forward rates	Cash Flow: Leg A	Cash Flow: Leg B	Cash Flow PV
$t = 0$	$D_0 = 1$	$F_0^{A,B} = S^{A,B}$	0	0	0
…	…	…	…	…	…
t	D_t	$F_t^{A,B}$	$c^A \times \Delta_t$	$c^B \times \frac{S^{A,B}}{F_t^{A,B}} \times \Delta_t$	$D_t \times \left(c^A - c^B \times \frac{S^{A,B}}{F_t^{A,B}}\right) \times \Delta_t$
…	…	…	…	…	…
$t = T$	D_T	$F_T^{A,B}$	$1 + c^A \times \Delta_t$	$(1 + c^B \times \Delta_t) \times \frac{S^{A,B}}{F_t^{A,B}}$	$D_t \times \left[(1 + c^A \times \Delta_t) - (1 + c^B \times \Delta_t) \times \frac{S^{A,B}}{F_t^{A,B}}\right]$

Under the FMV valuation, the sum of the cash flows PV values in the last column of the cash flow table must be equal to zero (**validation test**).

Swap Approximations (Based on Zero-Coupon Bonds)

The swap premium estimated for zero-coupon bonds can be used as a proxy for the swap premium estimated for bonds with generic coupon structure. The zero-coupon swap premium/discount equation derived in this section can also be used to derive implied forward rates.

Suppose that $c^A = 0, c^B = 0$ and P^A and P^B are respectively prices of zero-coupon bonds denominated in currencies A and B. Then the equation derived in the section above can be presented as follows

$$D_T = D_T \times \frac{P^A}{P^B} \times \frac{S^{A,B}}{F_T^{A,B}}$$

where $P^A = D_T$ is the price of zero-coupon bond denominated in currency A. The equation relates the value of one unit of cash received in period T and denominated in currency A to the same unit of cash, which is replicated by (i) converting the price P^A of the instrument into currency B; (ii) purchase of zero-coupon bond denominated in currency B; and finally (iii) using forward FX rate to convert the unit of cash denominated in currency B into a unit of cash denominated in currency A.

The above equation can be represented equivalently as

$$P^B = P^A \times \frac{S^{A,B}}{F_T^{A,B}}$$

If the currency forward rate is above the spot rate, $F_T^{A,B} > S^{A,B}$, then the zero-coupon bond denominated in currency B is traded at discount compared to the zero-coupon bond denominated in currency A ($P^B < P^A$).

The equation can be represented equivalently in terms of the yield rates:

$$\frac{1}{(1+y^B)^T} = \frac{1}{(1+y^A)^T} \times \frac{S^{A,B}}{F_T^{A,B}}$$

or

$$y^B = (1+y^A)\left(\frac{F_T^{A,B}}{S^{A,B}}\right)^{\frac{1}{T}} - 1 \approx y^A + \left[\left(\frac{F_T^{A,B}}{S^{A,B}}\right)^{\frac{1}{T}} - 1\right] = y^A + f_t^{A,B} \qquad \text{(swp.4)}$$

where

$$f_T^{A,B} = \left(\frac{F_T^{A,B}}{S^{A,B}}\right)^{\frac{1}{T}} - 1 \qquad \text{(swp.5)}$$

is T −period forward rate.

The equation can be used to (i) derive an approximation of the swap premium/discount (note that it requires only spot rates and forward rates at maturity term and does not require the full term structure of the forward rates); and (ii) derive implied forward rates with maturity T from zero-coupon yield rates.

Fixed-to-Float Swap

Swap Adjustment for Interest Payment Terms Differences

In addition to major differences between interest rates (such as currency and fixed/float type), the interest rates also differ in interest payment frequency and day count basis. Typically these adjustment are relatively small (a few basis points). The formulas below derive approximate adjustments for the terms differences.

Adjustment for Frequency

With higher frequency of interest payments, the bond holder has an option to reinvest the interest income and generate a higher return. Therefore an adjustment for interest payment frequency need to be performed generally. Let's illustrate it by example.

Suppose that interest rate on a bond with semi-annual interest payments is i (and principal amount is normalized to 1). Suppose that the bond terms are amended to annual frequency payment.

With semi-annual payments, the bond holder receives $\frac{i\%}{2}$ interest amount, which is reinvested then into a new asset for a six-month period. Suppose that the short-term reinvestment rate is i^*. Then additional return on the investment is $\Delta i = \frac{i \times i^*}{4}$.

For a general frequency parameter $f = 2, 3, 4, 6, 12$, the equation that converts frequency f into annual frequency is described as follows:

$$\Delta i(f) = \frac{i}{f} \times \left(\left(1 + \frac{i^*}{f}\right)^{f-1} - 1 \right) \tag{fq.1}$$

If the adjustment is performed from frequency f_1 to frequency f_2, then the adjustment is calculated as follows:

$$\Delta i(f_1, f_2) = \Delta i(f_1) - \Delta i(f_2) \tag{fq.2}$$

In general the refinancing rate i^* needs to be calculated separately. For simplicity, we use a one-year risk-free rate (estimated from the discount rates used in swap calculations) as a proxy for the refinancing rate.

Adjustment for Day Count

There are typically four major day count conventions used in the interest calculations: 30/360, Act/Act, Act/365, and Act/360.

The 30/360, Act/Act, and Act/365 day count basis correspond to approximately the same number of interest payment days during the calendar year (Act/365 day count basis has one extra interest payment day during the leap years with 366 calendar days). Therefore, no adjustment is performed for the day count basis if both legs of the swap have one of the three above day count conventions.

Act/360 day count basis has approximately 5-6 additional interest payment days during the calendar year. The adjustment in this case is performed as follows:

$$y^{non-Act/360} = y^{Act/360} \times \frac{Act}{360} \sim y^{Act/360} \times \left(1 + \frac{5}{360}\right) \sim y^{Act/360} + y^{Act/360} \times 1.4\% \qquad \text{(dc.1)}$$

For example, if $y^{Act/360} = 5\%$, then adjustment from Act/360 to non-Act/360 day count basis is equal approximately to 5% x 1.4% = 7bps.

Implied FX Rates

Overview

We discussed how the FX rates are related with the interest rates and how the cross-currency interest rate swap is estimated based on the FX rates. In this section we discuss the reverse problem: how to estimate the FX rates based on the observed parity between interest rates in different countries.

The estimation of implied FX rates is required if the FX rates are not observed directly. There are typically two types of interest rate parity that can be used to estimate the FX rates:

- **Basis swap curves**.
- **Sovereign debt in denominated in multiple currencies**.

If the FX rates are not directly available, then the interest rate swaps are estimated as follows.

1. Identify an interest rate parity using either basis swap curves or sovereign debt transactions denominated in multiple currencies;
2. Estimate terms structure curves estimated based on the observed interest parities;
3. Estimate implied FX rates based on the observed interest rate parity described by the pair of term structure curves;
4. Apply estimated FX rates to perform an arbitrary cross-currency swap estimation.

Estimation of Implied FX Rates

General Formulas

Suppose that c_T^A and c_T^B are interest rates on debt transactions that have comparable terms with exception of the debt currency. The two interest rate curves are estimated either (i) based on basis swap curves or (ii) samples of sovereign debt transactions.

The interest rate relationship was derived in the previous section and is described by the following equation:

$$c_T^B = \frac{1}{\left[\sum_{t=1,\ldots,T} D_t \times \frac{S^{A,B}}{F_t^{A,B}}\right]} \times \left(D_T \times \left(1 - \frac{S^{A,B}}{F_T^{A,B}}\right) + c_T^A \times \sum_{t=1,\ldots,T} D_t\right)$$

or equivalently

$$\left[\sum_{t=1,\ldots,T} D_t \times \frac{S^{A,B}}{F_t^{A,B}}\right] \times c_T^B = D_T \times \left(1 - \frac{S^{A,B}}{F_T^{A,B}}\right) + c_T^A \times \sum_{t=1,\ldots,T} D_t$$

After rearranging the terms, the equation can be represented as follows

$$D_T \times \frac{S^{A,B}}{F_T^{A,B}} \times (1 + c_T^B) = D_T \times \left(1 + c_T^A\right) + \sum_{t=1,\dots,T-1} D_t \times \left(c_T^A - c_T^B \times \frac{S^{A,B}}{F_t^{A,B}}\right) \qquad (2.1)$$

The above equation presents the relationship between period T FX forward rate and the FX forward rates estimated for periods $t = 1, \dots, T-1$. The equation is estimated sequentially using the underlying term structures of interest rates.

If we denote

$$f_t^{A,B} = \frac{F_t^{A,B}}{S^{A,B}}$$

then the equation can be equivalently represented as follows

$$f_T^{A,B} = \frac{D_T \times (1 + c_T^B)}{D_T \times (1 + c_T^A) + A_{t-1}} \qquad \text{(fx.1)}$$

Where

$$A_T = \sum_{t=1,\dots,T} D_t \times \left(c_T^A - \frac{c_T^B}{f_t^{A,B}}\right) = A_{T-1} + D_T \times \left(c_T^A - \frac{c_T^B}{f_T^{A,B}}\right) \qquad \text{(fx.2)}$$

Implied FX using basis swap curves

Under this approach, an interest relationship used to derive the implied FX rates is constructed based on the cross-currency basis curves and fixed-to-float swap curves denominated in local and foreign currencies. We illustrate the approach for the CAD and USD currencies.

Suppose that USSW curve represents the USD Libor-3M-to-USD fixed rate swap curve, CDSW curve represents CDOR-3M-to-CAD fixed rate swap curve and CDBS represents USD Libor-3M-to-CAD CDOR-3M base rate swap curve. (The USSW, CDSW, and CDBS are the Bloomberg curve tickers). The following relationship between USD fixed and CAD fixed rates can be derived. Suppose that an investor holds a contract that pays the USD Libor-3M return. Then the investor has the following two strategies:

- **USD fixed rate**: use USSW curve to convert the USD Libor-3M into equivalent fixed rate;
- **CAD fixed rate**: (i) use CDBS curve to convert USD Libor-3M into CAD CDOR-3M and (ii) use CDSW to convert CAD CDOR-3M into CAD fixed rate.

The above strategy equivalence can be described by the following equation:

USSW (USD) = (CDSW + CDBS/100) (CAD)

(Note that USSW and CDSW curves are reported in percentage points and CDBS curve is reported in basis points, where 1% = 100bps).

Swap curve examples

A few selected swap curves constructed by Blomberg are presented below.

- **USD float-to-fixed swap**. The swap curve is described by USSW Bloomberg swap curve with the following swap leg parameters:
  - Floating leg: Q, Act/360, US Libor-3M
  - Fixed leg: SA, 30/360
- **CNY float-to-fixed swap**. The swap curve is described by Bloomberg swap curve with the following swap leg parameters
  - Floating leg: Q, Act/360, Shibor-3M
  - Fixed leg: Q, Act/Act
- **USD float-to-CNY float basis swap**. The basis swap curve swaps basis rates in different currencies. The USD-to-CNY basis swap is described by CCBS Bloomberg swap curve with the following swap leg parameters.
  - USD Floating leg: Q, Act/360, US Libor-3M
  - CNY Floating leg: Q, Act/360, Shibor-3M

Implied FX based on bond prices

Under this approach, an interest relationship used to derive the implied FX rates is constructed based on the sovereign debt transactions denominated in two currencies. Since the transactions are issued by the same government, it is assumed that the FX effect is the only effect that explains the difference in the rates (in practice, the assumption may not be true).

The approach can also be used to validate the consistency between the forward rates and the bond prices denominated in a specific currency. Validation may be informative when there is a very material spread between bid and ask forward prices or to test that the bond is traded at premium/discount relative to comparable USD-denominated bond after adjusting for the differences in currency (verify consistency between bond and FX markets).

The swap calculations under the bond prices approach can be summarized as follows:

- Search for the bonds denominated in both currencies A and B. Typically sovereign bonds issued in both USD (currency A) and domestic (currency B) currencies are identified;
- Construct term structure for both samples of bonds denominated in currencies A and B;
- Estimate the implied FX rates using the equations described above

Alternatively, the implied FX rates can be estimated as follows: (i) convert the term structure into zero-coupon term structure; and (ii) estimate implied FX rates on the zero-coupon term structures denominated in currencies A and B (as described in equations (swp.4) and (swp.5)).

Note that the spread between the term structures, estimated in step two, can be used as a proxy for the FX swap premium/discount.

Swaps Excel API

The Excel API for the swap functions is summarized in the table below.

Function name	Function arguments	Function output	Function description
AC.finance.irb.fx.swap	CurveFX fwdCurve, CurveDiscounts discounts	CalculatorSwapFX	Constructs FX swap calculator using FX forward curve and discount curve as inputs. Calculator estimates slope and intercept coefficients, $a_{0,T}$ and $a_{1,T}$
AC.finance.swap.run	CalculatorSwapFX calculator, Object obj	Object output (an instance of Curve, Note or Sample object)	Runs swap calculations for a given object. The object is either a Curve, a Note, or a Sample. The swaps are performed then either for Curve yields, Note yields or Sample notes yields.
AC.finance.swap.cf	Object date, Number tenor, Number yield, Number principal, CurveFX fwdCurve, CurveDiscounts discounts	List<List<Object>>	Construct cash flows described in the exhibit presented in these notes and return the cash flows as an array object
AC.finance.irb.get	Object obj, String key, Map<String, Object> params	Object	Retrieve an output field from a given object using a given key. The retrieved output depends on the input object type: *CalculatorSwapFX*: key = *fxswap-output*: return swap calculator intercept, slope, or proxy swap arrays; Map<String, Object> params includes the following (key, value) pairs: *fxswap-param* => { *fxswap-intercept, fxswap-slope, fxswap-proxy*}