NPV VALUATION METHODS

TRANSFER PRICING APPLICATIONS

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List of Abbreviations

The following abbreviations and symbols are used in this guide:

AAF	Annuity adjustment factor
ac.finance.NPV	Excel/java based NPV valuation tool developed as part of this guide
CDS	Credit default swap
CUP	Controlled unrelated price
CUT	Comparable unrelated transaction
DRB	Discount rate benchmarking
FMV	Fair market value
PIK	Pay-in-kind
NPV	Net present value
OECD Guidelines	"BEPS Actions 8 – 10, Financial Transactions", a draft published in July – September 2018 for the purposes of public discussion
SWPM	Bloomberg interest rate swap tool
TP	Transfer pricing
WACC	Weighted-average cost of capital
YTM	Yield to maturity

Section 1: Introduction

The net present value (**NPV**) is a standard approach to estimate the value of a general cash flow. The methodology has multiple applications, some of which are listed below.

- ► **Company value**. Estimate the value of a company based on the company projected cash flow analysis;
- ► **Loan value**. Estimate the value of a loan. The loan valuation transfer pricing (**TP**) analysis is typically performed whenever a loan is transferred from one lender to another;
- ► **Loan provisions value**. Estimate the value of loan provisions such as for example loan amortization provision, interest deferral provision, and other;
- ► **Interest prepayment analysis**. The TP analysis of interest prepayment is performed when the company plans to prepay a certain amount of interest at a discount. The objective of the TP analysis is to estimate the arm's length value of the interest prepayment discount;
- ► **Swap valuation**. Application of the NPV methodology for interest rate swap valuation is discussed in detail in the related "swap valuation" guide;
- ► **Risk-neutral default probabilities**. Estimation of risk-neutral default probabilities is described in detail in the related "financial guarantee valuation" guide. The probabilities are estimated as part of Hull-White CDS valuation model to estimate arbitrary bond derivative instruments such as CDS contracts, financial guarantees, and other.

This guide described general principals of NPV methodology and its applications for loan valuation, loan provisions valuation, and interest prepayment analysis. The key difference of the NPV methodology from the methodology applied for option valuations is that NPV models assume deterministic cash flows and do not take into account the cash flows volatility. Option valuation requires to implement more complex stochastic models, in which volatility is one of the key factors which determines the option value.

1.1 Terminology

The following terminology is used throughout the guide.

- ► **Clean price**. Clean price is calculated assuming zero accrued interest amount. Quoted prices and respective yield rates are typically clean prices. Clean prices do not have discontinuity at the interest payment dates.
- ► **Dirty price**. Dirty price includes the value of the accrued interest into the loan value and represents the actual price of the loan.
- ► **Future price**. The future price is calculated as of the future date. The future price is used, for example, in bond forwards valuation (Black model) or in convertibility option valuation.
- ► **Par value**. Par value equals the sum of the loan principal and accrued interest amounts.
- ► **Zero-coupon bond**. The bond that does not make any coupon payments and pays only the face value at maturity. Zero-coupon values are typically traded at discount.
- ► **Bullet loan**. The bullet loan is a loan with no pre-payment or pay-on-demand options and no amortization provisions. The principal amount of a bullet loan is paid in full on the loan maturity date.
- ► **Zero curve**. Zero curve is a term structure of yields on zero-coupon bonds. Zero curve effectively represents a term structure of discount rates, which can be directly converted to discount factors.

- ► **Bootstrapping**. In finance, bootstrapping is a method for constructing a (zero-coupon) fixed-income yield curve from the prices of a set of coupon-bearing products, e.g. bonds and swaps.[1] Bootstrapping is applied to estimate zero curve for yields (based on the yield term structure of coupon-bearing bonds as discussed in Section 2.1) or interest forward rates (based on the swap curve. The bootstrap method for interest forward rates is not covered in this guide).
- ► **Discount factor**. Price of zero-coupon bond.
- ► **Risk-adjusted discount factor**. We distinguish between risk-free and risk-adjusted discount factors. Risk-free discount factors assume that the underlying zero-coupon bonds are default free. Risk-adjusted discount factors are adjusted for the default risk of underlying zero-coupon bonds.
- ► **Annuity adjustment factor (AAF)**. AAF is the NPV value of the cash flow that pays $1 in each period. The AAF is typically calculated to convert a fixed upfront price (cost) into a flow of periodic payments.
- ► **Yield-to-maturity (YTM) rates**. Yield rate is a constant discount rate which is estimated implicitly from the NPV equation for the loan price.[2] Yield rates depend on the underlying cash flow structure of the loan and are typically estimated for bullet non-amortized loans. The yield rates are described by the underlying bonds including the list of bond parameters such as currency, frequency, and day count basis of the underlying bonds. For example, US Treasury, semi-annual, 30/360 yields are estimated based on a sample of US Treasury bonds, which typically have semi-annual frequency and 30/360 day count basis.
- ► **Yield-to-next-call rates**. The yield-to-next-call rates are estimated similarly to the YTM rates but assuming that the loan (bond) is called on the first date when it becomes callable. The yield-to-next-call values provide a useful information on whether there is a material risk that the loan (bond) will be called.
- ► **Internal rate of return (IRR)**. The internal rate of return is a discount rate that makes the net present value (NPV) of all cash flows from a particular project equal to zero.
- ► **Hurdle rate**. The hurdle rate, also called the minimum acceptable rate of return (**MARR**), is the lowest rate of return that the project must earn in order to offset the costs of the investment.

[1] https://en.wikipedia.org/wiki/Bootstrapping_(finance).

[2] Since the NPV valuation is performed assuming deterministic cash flows and discounts, the process that determines future yield rates is also assumed to be deterministic. Alternative stochastic interest rate models are discussed in the related interest rate option valuation guide.

Section 2: NPV Valuation Methodology

One of the trade-offs when a specific NPV method is selected is between the simplicity and the accuracy of the NPV equations. Specifically the following options are available:

- **Discount factors**. Use exact or approximate discount factors.

  In a simplified NPV approach the discount factors are estimated based on a flat yield term structure. The yield rates are estimated for a single maturity term matching the maturity term of the cash flows and are applied for each cash flow payment. This approach produces a reasonably good approximation for example for non-amortized loans when the largest cash flow payment is made on the loan maturity date. For amortized loans this approach may not be suitable as it will produce a poor approximation of the actual loan value.

  In an alternative approach the full term structure of yield rates is estimated and the discount for each specific cash flow payment is calculated using the maturity matching the period when the cash flow payment is made.

- **Cash flows**. Use exact or approximate cash flow calculations.

  In a simplified approach the cash flows are calculated using approximate day count value estimated uniformly for each period between two consecutive cash flow payments. The assumption often allows to derive an exact formula for the NPV value.

  In an alternative approach, exact non-uniform cash flows are calculated. The approach allows only for a numeric estimation of the NPV value.

In addition to deriving NPV equations, we also briefly discuss how to validate the results of the NPV calculations. Validation is illustrated for the following examples.

- **Bullet Loan**. The following validation test can be performed for a bullet loan: if interest rate i on the bullet loan is set equal to $i = y_T$ (where y_T is the yield rate with maturity term matching loan maturity), then the loan NPV value must be equal to the loan clean price.

- **Interest deferral**. If the term structure of yield rates is flat and deferred interest is capitalized, then the NPV value of the deferral provision must be equal to zero. For an increasing term structure, the NPV value of the loan with interest deferral must be lower than the NPV value of the bullet loan. The yield rate on the loan with interest deferral must be adjusted downward to produce a yield on a comparable bullet loan.

- **Amortized loan**. If the term structure of yield rates is flat then the NPV value of amortized loan must be equal to the NPV value of the bullet loan (the value of amortization provision is zero). For an increasing term structure, the NPV value of the amortized loan must be higher than the NPV value of the bullet loan. The yield rate on the amortized loan must be adjusted upward to produce a yield on a comparable bullet loan.

2.1 Discount factors

By definition, discount factors are prices of zero-coupon bonds. Market discount factors are estimated based on a search of publicly traded bonds/notes with the terms matching the terms of the tested loan transaction. For the purposes of the NPV valuation we need to estimate the full term structure of discount rates and respective factors. Therefore the search for comparable bonds/notes does not have to be restricted by the tested loan maturity term.

The bonds/notes in the identified sample are typically not zero-coupon bonds and have regular coupon payments. For example, standard interest payment terms for the US bonds assume semi-annual frequency and 30/360 day count basis. After the term structure of the yield rates is estimated for the bonds with specific periodic interest payments, the yield structure needs to be converted into equivalent zero-curve.

2.1.1 Exact discount factors

The discount factors are estimated based on the following system of sequential equations.

For period t_1, the discount factor is estimated from the following equation

$$(1 + y_1\alpha_1)D_1 = 1$$

where y_j is the yield rate with maturity term t_j, α_j is the day count estimated for the period $[t_{j-1}, t_j]$, and D_j is the discount factor for period t_j. The discount factor for the first period is estimated as

$$D_1 = \frac{1}{1 + y_1\alpha_1}$$

Suppose now that $D_1, \dots, D_{i-1}$ discount factors are calculated for periods $t_1, \dots, t_{j-1}$. Then discount factor D_j for period t_j is derived from the following equation.

$$y_j\left[\alpha_1 D_1 + \cdots + \alpha_{j-1}D_{j-1}\right] + \left(1 + y_j\alpha_j\right)D_j = 1 \tag{2.1}$$

or

$$D_j = \frac{1 - y_j\left[\alpha_1 D_1 + \cdots + \alpha_{j-1}D_{j-1}\right]}{1 + y_j\alpha_j} \tag{2.2}$$

2.1.2 Discount factors based on flat yield term structure

For constant yield rates (flat yield term structure) equation (2.1) can be simplified as follows.[3]

$$D_j = \frac{1}{1 + y\alpha_1} \times \cdots \times \frac{1}{1 + y\alpha_j} = D_{j-1} \times \frac{1}{1 + y\alpha_j} \tag{2.3}$$

2.1.3 Discount factors based on flat yield term structure and constant day count

If the day count basis c_j is approximated by a constant day count $c_j = c$, then the formula can be further simplified and presented as

$$D_j = \frac{1}{1 + y\alpha_1} \times \left[\frac{1}{1 + y\alpha}\right]^{j-1} \tag{2.4}$$

where α_1 is the day count calculated for the first period (which may be not a full interest payment period). If α^* is annualized day count and f is interest payment frequency, then $\alpha_1 = \alpha^* \times t_1$, $\alpha = \frac{\alpha^*}{f}$, and $j-1 = f \times (t - t_1)$. Equation (2.3) is represented then equivalently as follows

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[3] The equation is derived as follows. Substituting constant y into (2.1), we obtain $y \times \left[\alpha_1 D_1 + \cdots + \alpha_{j-1}D_{j-1}\right] + \left(1 + y\alpha_j\right)D_j = 1$ and $y \times \left[\alpha_1 D_1 + \cdots + \alpha_{j-2}D_{j-2}\right] + \left(1 + y\alpha_{j-1}\right)D_{j-1} = 1$. Therefore $1 - D_{j-1} + \left(1 + y\alpha_j\right)D_j = 1$ or equivalently $D_j = D_{j-1} \times \frac{1}{1+y\alpha_j}$.

$$D_t = \frac{1}{1 + y\alpha^* t_1} \times \left[\frac{1}{1 + y\frac{\alpha^*}{f}}\right]^{f\times(t-t_1)} \quad (2.5)$$

For day 30/360, Actual/Actual, and Actual/365 day counts the annualized value of α can be approximated as $\alpha^*=1$. For Actual/360 day count the annualized value of α can be approximated as α.

To summarize, the process of discount factors estimation is described as follows.

1. Search for bonds/notes comparable to the tested loan;
2. Estimate yield term structure $\{y_j\}$ based on the identified sample of comparable bonds/notes;
3. Convert the yield term structure into equivalent zero curve and estimate respective discount factors using equation (2.2) or its simplified versions (2.3) or (2.4);

2.1.4 Selection of discount factor estimation approach

The following considerations should be taken into account when selecting a general or simplified approach to discount factors estimation.

- ► If the loan has interest deferral, amortization provision, or other material balloon payments prior to the loan maturity, then a generic approach to discount factors should be applied. Applying approximate yields based on flat term structure will generally result in material understatement of the loan FMV;
- ► If the loan is a bullet loan and interest benchmarking analysis is performed based on corporate note search (CNS) approach (so that estimation of full term structure is a time intensive exercise), a simplified approach based on flat term structure can be used. The simplified approach will produce reasonably accurate FMV values that can be validated using the Par test.

2.2 NPV equations

This section presents exact NPV value estimated numerically and approximate NPV equation estimated assuming flat yield term structure and constant day count.

2.2.1 Exact NPV calculations

In general case, when exact discount factors (described by equation (2.2)) and exact loan cash flows are estimated, the NPV is calculated numerically using the following equation

$$\mathrm{NPV} = \sum_{j=1}^{n} \mathrm{D_j} \times \mathrm{c_j} + \mathrm{D_n} \times \mathrm{P} \quad (2.6)$$

where c_j is loan coupon payment in period t_j and P is loan principal amount. Numeric valuation is also required if simplified flat yield term structure is applied and the coupon cash flows are calculated exactly.

2.2.2 Approximate NPV calculations

Equation (2.6) can be simplified in the case when the discount factors are estimated using equation (2.4).

2.2.2.1 AAF

Annuity adjustment factor is defined as a cumulative discount factor: $AAF_n = \sum_{j=1}^{n} D_j$. If discount factors are represented by geometric progression described by equation (2.4), the AAF equation can be simplified as follows.[4]

$$AAF_n = \frac{1+y\alpha}{1+y\alpha_1} \times \left(\frac{1 - \left[\frac{1}{1+y\alpha}\right]^n}{y\alpha} \right) = \frac{1}{y\alpha} \times \left(\frac{1+y\alpha}{1+y\alpha_1} - D_n \right) \tag{2.7}$$

where

$$D_n = \frac{1}{1+y\alpha_1} \times \left[\frac{1}{1+y\alpha}\right]^{n-1}$$

If the first period is full interest period, $\alpha_1 = \alpha$, then the equation is simplified as follows

$$AAF_n = \frac{1 - D_n}{y\alpha} \tag{2.8}$$

where

$$D_n = \left[\frac{1}{1+y\alpha}\right]^n$$

2.2.2.2 Loan NPV

If the cash flows used in the NPV calculations are represented by the cash flows of a bullet loan with regular interest payments, then the NPV value can be broken down into the NPV of the coupon payments and NPV of the principal repayment. The loan NPV value is described by the following equation

$$NPV = AAF_n \times (c \times \alpha \times P) + D_n \times P = P \times \left[\frac{1+y\alpha}{1+y\alpha_1} \times \frac{c}{y} + D_n \times \left(1 - \frac{c}{y}\right) \right] \tag{2.9}$$

If the first period is the full interest period, then equation (2.9) is simplified as follows

$$NPV = P \times \left[\frac{c}{y} + D_n \times \left(1 - \frac{c}{y}\right) \right] = P \times \left[\frac{c}{y}(1 - D_n) + D_n \right] \tag{2.10}$$

The term $P \times \frac{c}{y}(1-D_n)$ represents the NPV of the coupon payments and the term $P \times D_n$ represents the NPV of the principal repayment.

2.3 Yield rates

Loan yield value is estimated as implicit solution of equation (2.9) or (2.10) if the first period is the full interest period. Solution always exists and is unique because the NPV value decreases monotonously with the yield value y.

[4] Equation for the geometric progression with $x = \frac{1}{1+y\alpha}$ (and $1 - x = \frac{y\alpha}{1+y\alpha}$) is presented as follows: $x_1 \times (1 + x + ... + x^{n-1}) = x_1 \times \frac{1-x^n}{1-x} = \frac{1}{1+y\alpha_1} \times \frac{(1+y\alpha) \times \left(1 - \left[\frac{1}{1+y\alpha}\right]^n\right)}{ya}$.

The implicit solution can be estimated for example by running the binary search algorithm on a range of yields, where the range can be estimated as follows. Suppose that P^* is the fair market value of the loan.

Case $\frac{P^*}{P}<1$ (loan is valued under par, which corresponds to the solution $y>c$). The lower bound of the range is set at $y=c$. The upper bound of the range is selected so that $\frac{c}{y}\leq\frac{1}{2}\times\frac{P^*}{P}$ and $D_n\leq\frac{1}{2}\times\frac{P^*}{P}$. The range of yield rates is described by the following equation

$$\left[c,\quad \max\left(2c\times\frac{P}{P^*},\ \frac{1}{\alpha}\times\left(\frac{1}{\left(\frac{1}{2}\times\frac{P^*}{P}\right)^{\frac{1}{n}}}\right)-1\right)\right]$$

Case $\frac{P^*}{P}>1$ (loan is valued over par, which corresponds to the solution $\frac{P^*}{P}>1$). In general solution to the equation (2.10) can be either positive or negative. First, we derive conditions for the maximum value of $\boldsymbol{\frac{P^*}{P}}$ such that the yield rate corresponding to the price P^* is positive. If $y\to 0$, then $D_n\sim 1\text{-}y\alpha n$.[5] Therefore $\frac{c}{y}(1-D_n)+D_n\sim c\times\alpha\times n+1$. Therefore whenever

$$\frac{P^*}{P}>1+c\times\alpha\times n$$

a positive yield rate solution does not exist for the price $\boldsymbol{P^*}$**.** For example, if $\alpha=1$ and $n=1$, then the price of the loan P^* cannot exceed the $P\times(1+c)$ threshold, which corresponds to total undiscounted sum of principal and interest payments.

The upper bound of the range is set at $\frac{P^*}{P}>1$. The lower bound is set as a small number ε close to zero such that the NPV at $y=\varepsilon>0$ exceeds P^*.

Positive solution

$$[\varepsilon>0,\ c]$$

If $\frac{P^*}{P}<1+c\times\alpha\times n$, then the solution of the equation (2.10) is negative.

Negative solution

$$[???,\ \varepsilon<0,]$$

If $\frac{P^*}{P}=1+c\times\alpha\times n$ then $y=0$.

[5] $D_n=\left[\frac{1}{1+y\alpha}\right]^n=e^{-n\log(1+y\alpha)}=e^{-y\alpha n}\sim 1\text{-}y\alpha n$

Section 3: Ranges of NPV values

Sections below show how to derive the NPV value assuming that the market term structure of interest rates are modelled as a single term structure which is applied to derive the respective structure of discount factors. In practice, the market interest rates applicable to the tested loan are described by a range of term structures, which model a potential variation in the loan valuation results.

As the borrower / lender consider issuing a new loan (or refinancing existing loan) they take into account potential variation in the loan valuation numbers. In the case of a single term structure of interest rates (and assuming that interest rate equals to the discount rate), the loan is valued at par and both the borrower and the lender are indifferent regarding whether the loan is issued or not.

The purpose of this section is to show how a range of NPV values can be generated consistently with the NPV estimation principals so that the borrower and lender decide on a loan refinancing transaction (and respective compensation between counter-parties) based on the range of potential valuation results. The modelling principals should be selected so that the results of the analysis are consistent with the intuition. NPV range modelling is based on the following principals:

1. The ranges are generated due to discrepancy in the interest rates estimated for the refinanced transacting and the market yield rates (so that the loan value may deviate from the par value from the perspective of both the borrower and the lender);
2. The generated ranges of the NPV values satisfy the following regularity condition: the higher are the market yield rates, the larger (net) compensation should be provided to the borrower to enter into the loan refinancing transaction (the net compensation from borrower to lender is a decreasing function of the market yield rate);
3. The valuation of the NPV ranges is based on standard NPV analysis presented above, which is applied to the range of interest and discount rates.

Under the assumptions which produce a range of NPV values, the analysis should be performed from the borrower's and lender's perspective to ensure that both the borrower and the lender are better off in the restructuring transaction. In particular, conditions under which a set of equilibrium values exists (core is non-empty). The conditions for the **existence of a non-empty core** in different types of financial transactions are discussed in the "*FSTP_08_Loan Restructuring_Analysis_v2.docm*" guide[6].

3.1 NPV valuation and tax analysis

The equations in the previous sections were derived for the market discounts and prices, which can be interpreted as "**before-tax**" prices. If, for example, a lender makes a loan then the equations (2.9) and (2.10) describe the loan value for the investor assuming zero taxes on the interest income. This is the market price of the before-tax loan interest and principal cash flows that the investor receives for the loan price. Therefore, the fair-market value (FMV) of the loan must be performed as a "before-tax" valuation.

The before-tax prices are actual market prices which do not depend on specific tax exposure of each borrower or lender. The values assigned by individual borrowers/lenders will be different depending on the taxes paid by the economic agents. For example, two loans L1 and L2 may be identical from the market

[6] The guide focuses only on the existence of the core generated by the differential in discount rates applicable to the borrower and the lender valuation problem. It does not take into consideration other factors such as for example a differential in the FMV of the interest payments. Therefore, the results presented in the guide do not apply directly to the tax analysis discussed below. However, certain 'sufficient' conditions for the core existence in a tax analysis problem can be derived from the results presented in the guide.

perspective and have the same market price but the taxes paid by a lender on loan L1 may exceed the taxes paid on loan L2. As a result, the lender will assign a higher value to loan L2 than loan L1. However, the discrepancy in the value by individual lender does not affect the loans market price but determines only which loan will be acquired by the lender.

Due to the "**after-tax**" idiosyncrasy of each economic agent valuation, the decisions to refinance the loan generally but not always depend on the specific tax exposure of the agent. For example, if the market value of a loan is below the par value, it is always beneficial for the borrower to refinance the loan since the same loan can be borrowed at par value and the difference is the borrower's benefit. This decision does not depend on the taxes paid by the borrower (and respective tax deductions related to the interest expense). On the other hand, if the loan value is above the par value, it still may be beneficial for the borrower to refinance the loan if the funds can be borrowed at different terms which increase tax deductions.

"After-tax" valuation is performed to assess whether it is beneficial from the tax perspective for both the borrower and the lender to restructure the loan transaction. The "after-tax" valuation takes into account not only the change in the transaction market value, but also specific taxes paid by both the borrower and the lender. The equations applied in after-tax NPV valuation analysis are described in the section below.

3.1.1 After-tax valuation equations

The adjustment to the NPV equations in the after-tax valuation can be summarized by the following rule.

<u>"After-tax" valuation rule</u>. *Both the cash flows of the tested transaction and the discount rates must be adjusted by the applicable tax rate for both the borrower and the lender. The tax rate applicable to the borrower and lender is applied respectively to perform the valuation from the borrower's and the lender's perspective*.

Tax calculations generally differ for different transactions and may be asymmetric for the borrower and the lender. For example, in the OID notes, the tax expense is calculated for the lender on accrual basis regardless of whether the interest income was received by the lender or not. The borrower can claim tax deductions only when the actual interest expense was paid.

While taxes generate idiosyncrasies in the "after-tax" valuations, the "after-tax" values must still be related to the market prices and not create arbitrage opportunities for the borrower and the lender. For example, the "after-tax" valuation models must produce the "before-tax" values whenever the tax is set to zero.

The "after-tax" discounts are calculated by multiplying the yield rates by one minus tax-rate:

$$y_t^{after-tax} = y_t \times (1 - \tau) \tag{2.11}$$

where τ is applicable tax rate. The discount factors are calculated based on the modified par identities:

$$y_j \times (1-\tau) \times \left[\alpha_1 D_1^{after-tax} + \cdots + \alpha_{j-1} D_{j-1}^{after-tax}\right] + \left(1 + y_j(1-\tau)\alpha_j\right) D_j^{after-tax} = 1 \tag{2.1*}$$

or

$$D_j^{after-tax} = \frac{1 - y_j \times (1-\tau) \times \left[\alpha_1 D_1^{after-tax} + \cdots + \alpha_{j-1} D_{j-1}^{after-tax}\right]}{1 + y_j(1-\tau)\alpha_j} \tag{2.2*}$$

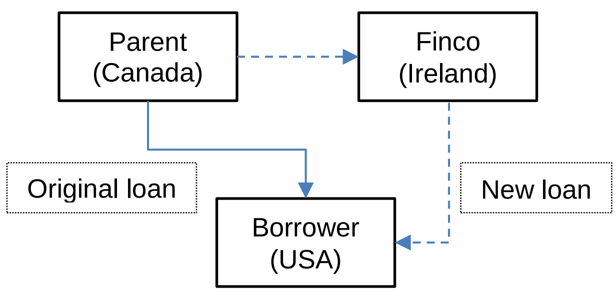
where D_1^{tax} is "after-tax" discount factor. Note that different "after-tax" discount factors are generally applied to the borrower and the lender.

The equations (2.1*) and (2.2*) can be justified as follows. Suppose that the transaction restructuring does not affect the applicable tax rates. Then the borrower's decision to refinance the loan whenever its price is below the par does not depend on the tax rates and interest expense schedule. Similarly, Similarly, the lender's decision to refinance the loan whenever its price is above the par does not depend on the tax rates and interest expense schedule. Therefore, the equation (2.1*) must hold for the borrower with the $\geq$ sign and for the lender with the $\leq$ sign. Assuming that the borrower's and the lender's valuation methods are the same for the same tax rates, equation (2.1*) must hold.

"After-tax" discounts described by the equation (2.2*) are applied also in the case when the transaction is restructured to get the benefit from the tax differential. Suppose that existing loan transaction can be refinanced by a lender at a lower tax rate. Then the difference between the "after-tax" interest expense cash flows represent a benefit to the lender which is evaluated using "after-tax" lender's discount factors.

3.1.2 After-tax valuation example: loan refinancing

Example is illustrated by the following diagram.



In the example, the loan between Canadian parent and US borrower is refinanced and replaced with the new loan from Finco to borrower. The objective of the loan refinancing is to reduce the income tax paid by the lender(s). The Lender's discount rate is estimated based on the Parent's tax rate (assuming that the Parent raises the funds in the market and therefore the financing costs are adjusted by the Parent's tax rate). In the loan refinancing, interest income is redirected from Borrower to Parent into the *interest income from Borrower to Finco. The Finco entity is selected assuming that the income tax rate applicable to Finco is lower than the income tax rate applicable to Parent.

FMV analysis comments:

1. Discount rates for the original and new loans are estimated as bullet rates.
2. The FMV of the original loan is estimated based on the actual interest rate and should be adjusted for the call/put option FMV (if present).
3. The FMV of the new loan is estimated based on the range of bullet discount rates and matching range of arm's length interest rates. If the interest rates are adjusted for the put/call option, the FMV should also be adjusted for the put/call option FMV (to produce the par value).
4. From the borrower's perspective, the discount rates and interest expense are adjusted by the same borrower's tax rate. From the lenders' perspective, the discount rates are adjusted by the Parent's tax rate and the interest income is adjusted by the Parent's tax rate for the original loan and by Finco's tax rate for the new loan. The interest rate differential generates a tax benefit for the lender (on consolidated basis) in the loan refinancing.

5. The taxed interest income paid by the borrower to Finco is distributed as tax-free dividends to the Parent. Tax-free dividend distribution is the key assumption which supports the tax gain to the lender (on consolidated basis) from the loan refinancing.
6. The FMV of the new loan is adjusted by the taxes paid (tax benefits) from the original loan repayment (in addition to the tax applied to the new loan cash flows).[7] The FMV of the original loan is estimated using tax-adjusted loan cash flows and tax adjusted discount rates.
7. The 'after-tax' fees ('net after-tax fee payable by borrower / acceptable by lender') are converted to the 'before-tax' fees ('gross-up fee payable by borrower / acceptable by lender').
8. Since the borrower's tax rate in the original and new loans is the same, the after-tax FMV of the new loan will be below par (par value minus the tax benefits). If the lender's tax rate on the new loan is lower than the tax rate on the original loan, the par value of the new loan FMV will be adjusted upward for the tax benefit and downward for the taxes paid on the original loan interest income. The aggregate impact is typically will be the FMV of the new loan exceeding the par value.

3.2 WACC discount rate

One of conceptual decisions that need to be made in the NPV analysis is whether to use refinancing rate or WACC as a discount rate. The decision depends on the conceptual approach to transfer pricing analysis. Specifically,

1. If the analysis is performed purely from 'market' perspective, then refinancing rate should be applied;
2. If the analysis takes specific facts and circumstances that apply to the tested entities, then WACC discount factor should also be considered as an option.

To illustrate specifically what is implied by 'market' perspective, suppose that the NPV analysis is performed for a loan restructuring analysis (for example, interest payment deferral / prepayment analysis). The restructured loan is priced from the perspective of the instrument which is traded in the market. The price of the loan depends on the market characteristics such as default risk, prepayment risk, term premium, and other but does not depend on specific business purpose of loan restructuring such as for example how the funds from the restructured loan will be used. The pricing method of a restructured loan is implemented as follows:

(i) Construct the market term structure of the yield rates applicable to the loan based on the loan credit risk, maturity term, and other terms and conditions;

(ii) Perform bootstrap procedure for the estimated term structure to derive discount factors;

(iii) Perform NPV analysis of the loan.

First two steps of the above algorithm estimate complete set of arbitrage-free prices which can be applied arbitrary instruments with comparable credit risk factor. The '**arbitrage**' pricing approach does not depend on any entity-specific facts which may require certain subjective judgement in the valuation analysis.

Application of WACC as a discount rate is based on the argument that the estimated cash flows are evaluated from the perspective of alternative use of funds ('**opportunity cost**' of funds). Since the opportunity cost is subjective in many cases, WACC is applied as a generic measure of the company cost

[7] If the original loan is an OID note, then the borrower's FMV is reduced by the tax on the accrued interest (tax cost is realised by the borrower only when the interest is paid) and the lender's FMV is adjusted by the latest quarter tax cost (lender pays taxes on quarterly interest schedule, not when the interest is actually paid).

of capital. Note that WACC depends not only on risk characteristics of the company (which are generally priced by market) but also on company-specific characteristics such as for example the company capital structure. Therefore, unlike the refinancing rate which is based on pure market arbitrage principal, WACC is not a pure market measure but also depends on the company capital structure decisions. WACC is typically estimated as a single value (and therefore does not include the term premium component). Therefore, WACC may not be a good measure to evaluate cost / benefit of intertemporal allocation of cash flows (as for example the case in the interest payment deferral / prepayment analysis).

WACC is typically applied to perform valuation new projects, assuming that specific capital (debt / equity structure) raised to fund the project matches the capital structure of the company.

Section 4: Validation of NPV estimation results

This section discusses the validation tests for the NPV valuation results applied for bullet loan, interest deferral, and interest amortization.

4.1 Bullet loan (Par test)

4.2 Amortized loan

4.3 Interest deferral

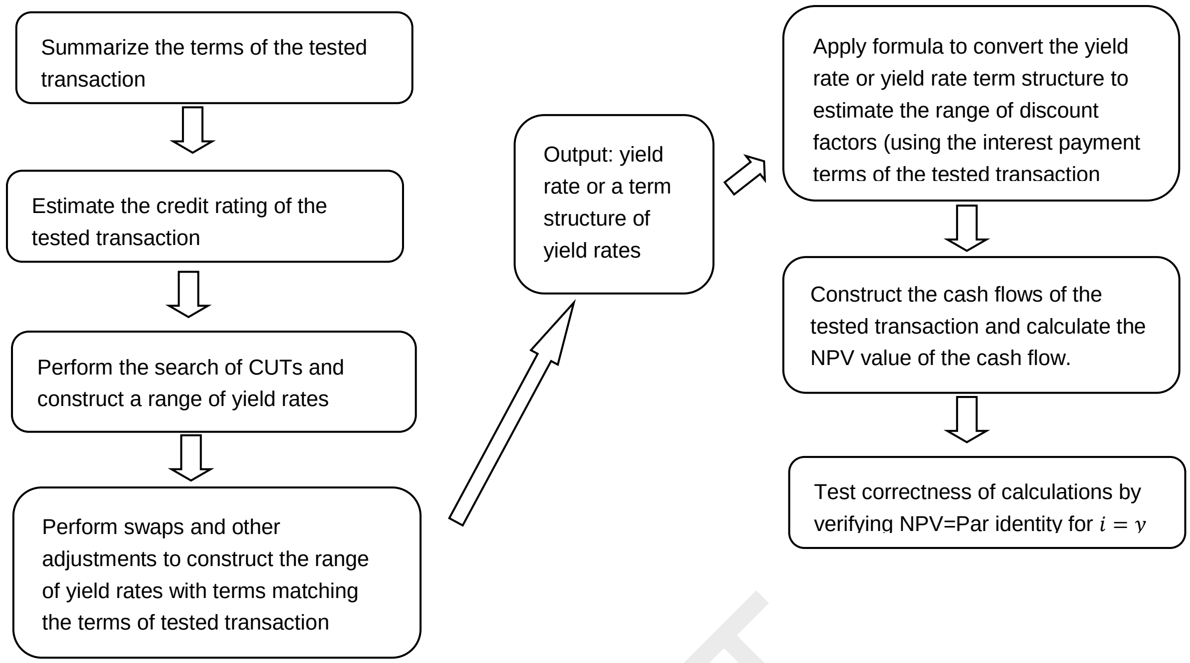
Section 5: Summary

NPV valuation process can be summarized as follows. The process is described for the loan NPV valuation but can be extended directly to a generic cash flow model.

1. Summarize material terms of the loan including
   - ► Loan currency and principal amount;
   - ► Loan maturity term;
   - ► Interest rate type (fixed, floating, or variable);
   - ► Interest payment terms (frequency and day count basis);
   - ► Loan options (prepayment or pay-on-demand);
   - ► Loan provisions (amortization, interest deferral, PIK provision);

   The loan cash flow payments are determined by the terms of the loan agreement.
2. Perform credit rating analysis of the loan for the purposes of discount rate benchmarking (**DRB**) analysis;
3. Perform DRB analysis. The analysis includes the following steps.
   - ► Search for comparable corporate notes/bonds with matching credit rating, industry sector, and other parameters (if necessary);
   - ► Adjust the yield rates of the identified sample of comparable bonds to construct a term structure of bullet yields;
   - ► Estimate discount factors using equations described in Section 2.1.
4. Construct the cash flows of the loan transaction;
5. Estimate the loan NPV value based on the constructed cash flows and estimated discount factors;
6. Validate the results of the analysis as discussed in Section 2.3.

The summary is illustrated by the following diagram.


Summarize the terms of the tested transaction
Estimate the credit rating of the tested transaction
Perform the search of CUTs and construct a range of yield rates
Perform swaps and other adjustments to construct the range of yield rates with terms matching the terms of tested transaction
Output: yield rate or a term structure of yield rates
Apply formula to convert the yield rate or yield rate term structure to estimate the range of discount factors (using the interest payment terms of the tested transaction
Construct the cash flows of the tested transaction and calculate the NPV value of the cash flow.
Test correctness of calculations by verifying NPV=Par identity for $i = y$

Appendix A: Transfer Pricing Classification of NPV Method

The NPV methods are traditionally referred to as the "**Other Methods**" under the OECD guidelines transfer pricing methods classification. From the perspective of the financial theory, NPV valuation is based on no-arbitrage principals, which are closely related to the controlled unrelated price (**CUP**) method under the OECD guidelines.

This section discusses the relation to the arbitrage valuation and CUP method. However since the relation to the CUP method is indirect, the "Other Methods" may still be a more appropriate reference for the NPV valuation methods.

A.1 NPV method as arbitrage-free valuation

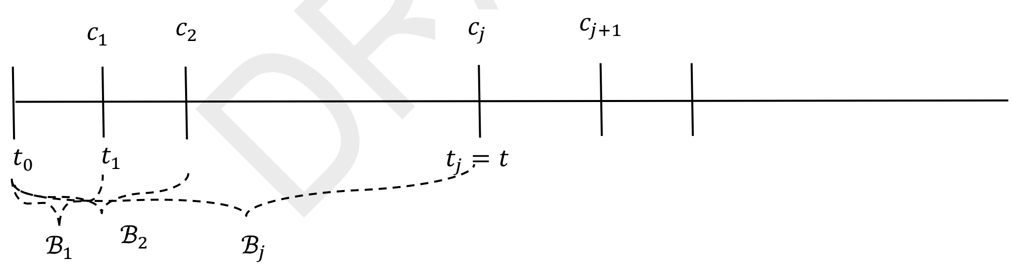
As discussed in Section 2, the NPV valuation is based on the following equation

$$NPV = \sum_{t=1,\dots,n} D_t C_t$$

where D_t is the discount factor and C_t is the cash flow in period t. By definition, discount factor are the prices of zero coupon bonds, which pay $1 dollar in period t or zero otherwise.[8] Suppose that $\mathcal{B}_j$ denotes a zero-coupon bond. Then the NPV equation represent the price of the following portfolio

$$\mathbb{C} = C_1\mathcal{B}_1 + C_2\mathcal{B}_2 + \cdots + C_t\mathcal{B}_t + \cdots$$

where the portfolio $\mathbb{C} = \left(C_1, C_2, \dots, C_j, C_{j+1}, \dots\right)$ replicates the cash flows of the underlying security. Replication of the cash flows with the portfolio of zero-coupon bonds represents an arbitrage approach to the cash flow valuation. Schematically the cash flow replication portfolio can be presented by the following diagram



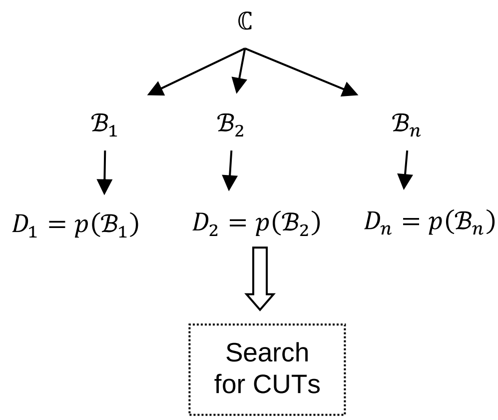
A.2 NPV method as a CUP approach

Estimation of the discount factors is the key element of the NPV methodology. As discussed above, by definition discount factor is a price of a zero-coupon bond $\mathcal{B}_t$. Therefore estimation of the discount factors is based on (i) the estimation of the credit rating and maturity term of the cash flows and (ii) searches for comparable bond transactions with similar credit profile and maturity term. Schematically it can be represented as follows.

In the diagram a general cash flow is represented as a portfolio of zero-coupon bonds with the default risk matching the default risk of the cash flows and maturity term ranging from zero to the maturity term of the cash flows. The yield rate of each zero-coupon bond is estimated then based on the searches for

[8] More generally, the NPV equation may be estimated using risk-adjusted discount factors, which represent the prices of risky zero-coupon bonds. A risky zero-coupon bond pays $1 in period t conditional on no default event and zero otherwise.

comparable unrelated transactions (**CUTs**). The yield rates are converted into related discount factors as discussed in Section 2.2.



As the diagram demonstrates, valuation of the discount factors is effectively performed using a CUP method.

Appendix B: Technical Implementation of NPV Methods

This section coves various practical aspects related to the implementation of the NPV valuation methods.

B.1 Forward discounts and prices

In most cases the valuation date[9] of a loan (or other financial transaction) is different from the actual loan issue date. The period $t = 0$ in the discount calculation equations (2.1) – (2.5) refers to the valuation date of the NPV analysis. Informally, the estimated discounts can be estimated using the curve data as of the valuation date and applied assuming that period $t = 0$ is the issue date of the loan.

A technically more correct approach is to assume that the loan is a forward instrument. The valuation as of the curve date is performed for a future loan issue date. The spot discount factors D_t estimated using equations (2.1) – (2.5) are converted into forward discount factors as follows.

Suppose that $\mathrm{D}_{0,\mathrm{t}}$=$\mathrm{D}_{\mathrm{t}}$ is a spot discount factor. Then the forward discount factor estimated as of future period s is estimated as follows.

$$D_{s,t} = \frac{D_{0,t}}{D_{0,s}} \tag{B.1}$$

The equation (B.1) for the forward discount factor is based on the following arbitrage argument. There are two alternative strategies to replicate a payoff of a security which pays $1 in period t (and zero otherwise).

- **Strategy A**. Purchase a zero-coupon bond with maturity t;
- **Strategy B**. Purchase a forward zero-coupon bond, which is issued in period s and matures in period t. By definition the price of the bond is $D_{s,t}$. Contemporaneously purchase a zero-coupon bond with maturity s and principal amount $D_{s,t}$. The principal amount $D_{s,t}$ received in period s is used to pay for the forward zero-coupon bond. The price of the zero-coupon bond with maturity s is $D_{0,s} \times D_{s,t}$, which must be equal to $D_{0,t}$.

The forward discount rates are applied not only to evaluate the loan clean (dirty) price at the loan issue (transfer) date but also at any future period of time. Future loan prices are used for example in Black model.[10]

B.2 Adjustment for options

If a loan (bond) is callable, puttable, convertible, sinkable, has a PIK provision or other options or provisions, they must be taken into account when performing the loan NPV analysis. In this section we illustrate how the NPV analysis is performed for a callable bond.

Suppose that an intercompany loan is transferred from one lender to another and the loan can be prepaid at any time with no penalty or make-whole provisions at an x-day notice period. Then the fair market value (FMV) of the loan is estimated as follows.

- Estimate the NPV value of the bullet loan (assuming the loan is held until maturity);

[9] The valuation date is sometimes also referred to as the curve date (the curve date terminology is used for example in Bloomberg swap valuation tool).

[10] https://en.wikipedia.org/wiki/Black_model

- ► Estimate the NPV value of the loan assuming that the loan is called immoderately and settled after x-day notice period.
- ► Estimate the FMV of the loan as the minimum of the two values:

The FMV of the loan can be summarized by the following equation

$$\mathrm{FMV}=\min\left[\mathrm{NPV}^{\mathrm{bullet}},\mathrm{NPV}^{\mathrm{called}}\right]$$

where NPV^{bullet} is the NPV of the bullet loan, and NPV^{called} is the NPV of the loan which is called immoderately and settled after x-day notice period.

B.3 Floating rate loans

If the loan has floating rate interest payments calculated as a base rate plus spread, then the future base rates must be estimated to calculated loan's projected coupon payments. There are several approaches to address the problem.

- ► Convert the floating rate into equivalent fixed rate and apply the fixed rate to estimate the NPV value;
- ► Estimate projected base rates (using for example economic projections of the interest rates published by large banks, policy institutions or other commercial or government organizations;
- ► Estimate forward base rates based on respective floating-to-fixed swap curve.

The last approach is our default approach to perform NPV valuation of the floating rate loans. Effectively the approach assumes that the floating rates are hedged by respective interest rate forward contracts and at each future date the uncertain floating rate coupon is converted into certain fixed coupon based on the respective terms of an interest rate forward contract.

If parameters of the swap curve floating leg match exactly the interest payment terms of the loan, then the swap curve can be applied directly to convert floating coupons into equivalent fixed coupon payments. For example, if the floating leg of a floating-to-fixed swap curve has quarterly frequency and actual/360 day count, which match the frequency and day count of the loan, then the floating rate of the loan can be replaced with equivalent fixed rate (= swap rate + spread) and the parameters of the fixed leg of the swap curve (e.g. semi-annual frequency and 30/360 day count) are applied in the loan NPV valuation.

An alternative more generic and formally more accurate approach is to estimate the forward rates for the base rate using Bloomberg floating-to-fixed interest rate swap (**SWPM**) tool. As part of interest rate swap valuation output, SWPM tool reports the forward rates for a given base rate selected on the floating leg of the swap tool (the forward rates are referred in the SWPM tool as the **reset rates**). The floating coupon payment in the NPV calculations is replaced then with the fixed rate = reset rate + spread, where the reset rate is estimated for the same date as the respective base rate used in floating coupon calculations. The approach can be summarized as follows.

- ► Perform a floating-to-fixed swap calculation using Bloomberg's SWPM swap tool. Select the base rate on the floating leg of the SWPM tool matching the base rate applied in the loan floating coupon calculations. Select the interest payment dates in the SWPM tool to match the interest payment dates of the loan;
- ► Go to the tab with the rest rates of the SWPM tool. Use the reset rates as forward rates for the base rate. Estimate each floating coupon rate (equal to base rate + spread) with the fixed coupon rate (equal to reset rate + spread, where reset rate is selected at the same date as the base rate);

- ► Perform the NPV calculations using estimated reset rates and respective equivalent fixed coupon rates of the loan.

Note that in theory the forward reset rates used by Bloomberg can be estimated based on the respective swap curve (by applying bootstrapping methods to the swap curve). However this functionality is currently not implemented in the ac.NPV tool. Effectively the SWPM tool is applied to perform bootstrapping of the respective floating-to-fixed swap curve and bootstrapping output (in the form of reset rates) is used as inputs in the ac.NPV tool.

B.4 Day count calculation rules

This section provides a technical description of the rules applied to estimate day count values used in the loan interest calculations. A day count rule must be additive with respect to dates: if $date_1 \leq date_2 \leq date_3$ are three arbitrary dates, then *day count($date_1$, $date_3$) = day count($date_1$, $date_2$) + day count($date_2$, $date_3$)*. The additivity property is applied for example to perform actual/actual or 30/360 day count calculations (as described below).

- ► **Actual/360**. The day count factor is calculated as the actual number of days between day one and day two divided by 360;
- ► **Actual/365**. The day count factor is calculated as the actual number of days between day one and day two divided by 365;
- ► **Actual/Actual**.[11] The denominator in the actual/actual day count calculations is either 366 or 365 depending whether a year is the leap year or not[12]. Therefore it may be necessary to break down the period between $date_1$ and $date_2$ into multiple periods to ensure that each pair of dates (in the period break down) is within the same calendar year (so that the denominator 365 or 366 is applied correctly for each pair of date). The day count for $date_1$ and $date_2$ is calculated then as a sum of day counts estimated for each date pair in the [$date_1$, $date_2$] period break down.

  The rule is illustrated by the following example. Suppose that the actual/actual day count is calculated for the 1-Dec-2016 to 31-Jan-2017 period. The denominator applied to the days in 2016 (which is a leap year) is 366 and denominator applied to the days in 2017 (which is not a leap year) is 365. Therefore the period 1-Dec-2016 – 31-Jan-2017 must be broken down into two periods: 1-Dec-2016 – 31-Dec-2016 and 1-Jan-2017 – 31-Jan-2017 and the day count is calculated as the sum of the day counts calculated for each of the two periods. Denominators 366 and 365 are applied respectively to the first and second periods.
- ► **30/360**.[13] 30/360 (ISDA) day count: factor is estimated as follows.

$$\alpha = \frac{360 \times (Year_1 - Year_0) + 30 \times (Month_1 - Month_0) + (Day_1 - Day_0)}{360}$$

  where $(Year_0, Month_0, Day_0)$ are year, month, and day of $\mathfrak{D}_0$ and $(Year_1, Month_1, Day_1)$ are year, month, and day of $\mathfrak{D}_1$. The following transformations of $\mathfrak{D}_0$ and $\mathfrak{D}_1$ are performed prior to calculating the day count using the above formula.

  - ► If $\mathfrak{D}_0$ is the last day of the month, then change $\mathfrak{D}_0$ to 30;
  - ► If $\mathfrak{D}_1$ is the last day of the month (unless $\mathfrak{D}_1$ is the maturity date and month of $\mathfrak{D}_1$ is February), then change $\mathfrak{D}_1$ to 30.

[11] http://en.wikipedia.org/wiki/Day_count_convention#Actual.2FActual_ISDA.

[12] A year is a leap year, if it is divided by 4. For example, 2012, 2016, 2020, etc. are leap years.

[13] http://en.wikipedia.org/wiki/Day_count_convention#30E.2F360_ISDA.

Suppose that period duration dt (measured in years) is calculated as $\mathrm{dt}=\frac{\mathrm{Actual}}{365}$. Then for medium- and long-term maturities the day count can be approximated as follows:

- Actual/Actual and 30/360 day count factors are approximated with $\mathrm{dt}\times\frac{365.0}{365.25}$;
- Actual/365 day count factors are approximated with dt;
- Actual/360 day count factors are approximated with $dt \times \frac{365.0}{360.0}$.

Appendix C: ac.finance.NPV Tool

Appendix D: Examples

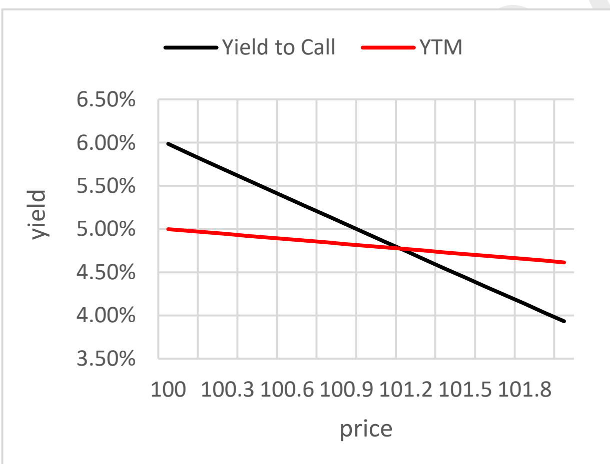
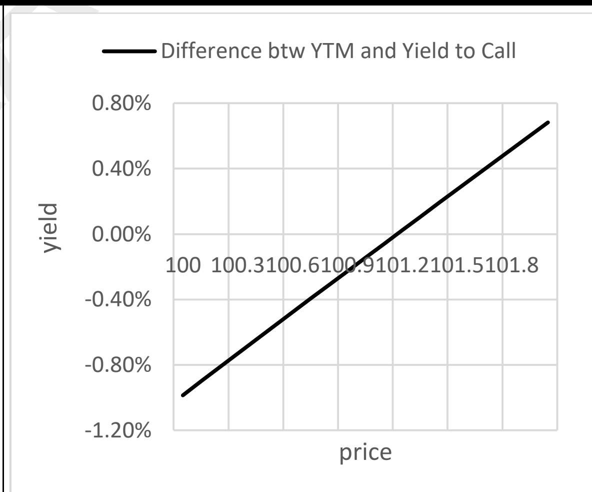
This section described several examples of the NPV valuation application, which can be observed in a transfer pricing project.

D.1 Prepayment risk in yield sample

Prepayment risk refers in this section to the case when the bond traded in the market is expected to be called in the near future. The yield-to-maturity (YTM) for this bond may not be representative of the bond actual yield since the prepayment risk moves the fair market value of the loan down to the par value (as it is expected to be prepaid) and respectively moves the market yield in the direction of the coupon rate.

The notes with prepayment risk should be reviewed and potentially removed from the sample as their YTM value are not representing properly the actual yields on the transactions. To identify such transactions, the YTM values can be compared to the yield-to-next-call values. A large spread between the two values indicated that the prepayment risk on the transaction is material.

The prepayment risk is illustrated by the following numeric example. The example is modelled using the following parameters: (i) coupon rate – 5%; (ii) duration to the next call date – one year; (iii) maturity term – five years; (iv) redemption price – 101. The YTM and the yield-to-next-call (and the difference between the two values) are presented for the bond prices ranging from the par value to 102 price.

YTM and Yield-to-next call values	Difference between YTM and Yield-to-next call values
	

The diagram shows that with the increase in the bond price (and respective increase in the prepayment risk) the gap between the YTM and the yield-to-next-call values also increases.

The prepayment risk is illustrated by the following real market data obtained through Bloomberg database. Both callable issuances in the example are USD denominated with standard semi-annual 30/360 interest payment terms. The valuation of the prepayment risk was performed as of 15 August 2017.

Ticker	Issuer Name	Maturity Date	Next Call Date	Redemption Price	Price	Coupon rate	YTM	Yield to Next Call	Yield Spread
EJ480374 Corp	BROOKFIELD RESIDENTIAL	15-Dec-20	15-Dec-17	101.625	102.972	6.50	5.509	2.363	3.145
EJ726235 Corp	BROOKFIELD RESID PROPERT	1-Jul-22	1-Jul-18	103.063	104.784	6.125	5.006	3.982	1.024

In the example, the first issuance has a shorter maturity but higher YTM value. The higher YTM value is explained by higher prepayment risk of the issuance, which pulls the YTM value in the direction of the 6.5% coupon rate on the note. Examples shows that prepayment risk exists for both transactions but is more significant for the first transaction, which explains a high YTM value on the note.

The YTM values in the prepayment risk analysis are estimated using standard Excel YIELD function. The results produced by the YIELD function are consistent with the yield values reported by Bloomberg (5.509% vs 5.506% YTM for EJ480374 Corp note and 5.006% vs 5.006% for EJ726235 Corp note).[14]

Two strategies can be applied to account for the prepayment risk in the sample.

- Remove the notes from the sample whenever the notes are priced above par and yield-to-next call is smaller than par value;
- Replace the YTM values with the yield-to-next call values and respectively replace maturity terms with the durations to the next call.

Alternatively, a combination of the two methods above can be applied to account for the prepayment risk in the yield sample.

D.2 Valuation of amortized Loans

An amortized loan is a loan with scheduled periodic payments that are applied to both principal and interest. An amortized loan payment first pays off the relevant interest expense for the period, after which the remainder of the payment reduces the principal.

Effective maturity of amortized loan is shorter than the stated maturity. Therefore, the term premium on amortized loan is typically lower compared to an equivalent non-amortized loan. The amortization adjustment is performed using NPV calculations as described below.

1. **Yield rate term structure**. Estimate term structure of the US$ yield rates with credit rating matching credit rating of the tested loan;
2. **Yield rate on tested loan**. Estimate the fixed US$ yield rate on the tested loan prior to amortization adjustment.
3. **Adjustment to par**. Multiply the term structure by a fixed factor to ensure that the non-amortized loan with the coupon rate estimated at step 2 is priced at par. Note that if the coupon rate on the non-amortized loan is set equal to the discount rate, the value of the non-amortized loan must be automatically equal to par (fixed factor = 1.0);
4. **Discount factor term structure**. Estimate term structure of discount rates;

[14] The yield to next call is estimated differently by Bloomberg compared to the results produced by YIELD function. Bloomberg reports 9.936% yield-to-next-call for EJ480374 Corp note (which is counter-intuitive) and 3.310% for EJ726235 Corp note. A similar to Bloomberg 3.345% yield-to-next-call for EJ726235 Corp note is produced by YIELD function if the next call date is set at 12-Sep-17 (approximately one month after the valuation date) and the applicable redemption price is equal to 104.594 (which was effective after 1-Jul-17).

5. **Amortized loan yield**. Estimate the coupon rate on the amortized loan so that it is priced at par;
6. **Amortization discount adjustment**. Amortization discount adjustment equals to the difference between (i) yield rate estimated in step 2; and (ii) the yield rate on amortized loan estimated at step 5.

Not that after the adjustments both amortized and non-amortized loans are priced at par. As a high-level **proxy**, the yield rate adjustment on amortized loan can be estimated as follows:

1. Estimate effective maturity term of the amortized loan as $t^* = \sum_i t_i \times \alpha_i$, where α_i is the percentage of the loan principal amortized in period t_i;
2. Calculate (adjusted) market yield rates as of effective and stated maturity terms of the amortized loan;
3. Calculate the amortization discount adjustment as the difference between the yield rates estimated for the(adjusted) stated and effective maturity terms.

The high-level proxy adjustment should be reasonably close to the adjustment estimated using the NPV valuation approach.

Note that adjustment of the market yield curve to ensure that the tested amortized loan is valued at par is a necessary step to produce intuitively reasonable numbers for amortization adjustment. If alternatively, the amortized loan is valued significantly different from par (under current market yield term structure) and the interest rate on non-amortized loan is adjusted to produce the NPV of non-amortized loan equal to the NPV of amortized loan, the estimated amortization adjustment will generally be inconsistent with intuition (and in some cases even negative).

D.3 Valuation of loans with interest deferral and prepayment

Appendix E: References