INTEREST RATE SWAP VALUATION
A TECHNICAL GUIDE

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List of Abbreviations

The following abbreviations and symbols are used in this guide:

ac.finance.swap	Excel/java-based interest rate swap valuation tool developed as part of this guide
BA	Bankers' acceptance
Bloomberg	Bloomberg Professional Services
BoC	Bank of Canada
FRA	Forward rate agreement
FX	Foreign exchange
Libor	London Inter-bank Offered Rate
NPV	Net present value
OECD Guidelines	"BEPS Actions 8 – 10, Financial Transactions", a draft published in July – September 2018 for the purposes of public discussion
OTC	Over the counter
Prime	Prime rate
SOFR	Secured Overnight Financing Rate
STIR futures	Short-term interest rate futures
SWPM	Bloomberg interest rate swap tool

Section 1 Introduction

Interest rate swaps are one of the most routine valuation tasks in transfer pricing analysis of financial transactions. One of the most frequent applications of swap valuation analysis is to convert cash flows denominated in one currency into cash flows denominated in another currency. For example, if tested loan is denominated in CAD and comparable bonds are denominated in USD, a cross-currency USD-to-CAD swap has to be estimated to convert the USD-denominated yield rates on the comparable bonds into equivalent CAD-denominated yield rates.

A standard swap valuation is performed to adjust for the following differences in the cash flows of two different bond/loan transactions:

- Currency;
- Interest rate type (fixed or floating);
- Interest rate payment frequency;
- Day count basis (Actual/Actual, 30/360, Actual/360, Actual/365).

The swap valuation can be also applied to adjust for other differences such as (i) loan amortization; (ii) variable step coupon; (iii) interest deferral or PIK provision, and other. Some of the above swap valuation models are discussed in the respective "Interest Benchmarking Analysis" guide. The focus of this guide is to present in detail the valuation methods for cross-currency swap transactions.

1.1 Swap definition

Definition:[1] A **swap** is a derivative contract where two parties exchange financial instruments. Most swaps are derivatives in which two counterparties exchange cash flows of one party's financial instrument for those of the other party's financial instrument. Specifically, two counterparties agree to exchange one stream of cash flows against another stream. These streams are called the legs of the swap. The swap agreement defines the dates when the cash flows are to be paid and the way they are accrued and calculated. Usually at the time when the contract is initiated, at least one of these series of cash flows is determined by an uncertain variable such as a floating interest rate, foreign exchange rate, equity price, or commodity price.

The cash flows are calculated over a notional principal amount. Contrary to a future, a forward or an option, the notional amount is usually not exchanged between counterparties. Consequently, swaps can be in cash or collateral. Swaps are among the most heavily traded financial contracts in the world: the total amount of interest rates and currency swaps outstanding is more than $348 trillion in 2010, according to Bank for International Settlements (BIS).

Most swaps are traded over the counter (OTC), "tailor-made" for the counterparties. Some types of swaps are also exchanged on futures markets such as the Chicago Mercantile Exchange, the largest U.S. futures market, the Chicago Board Options Exchange, Intercontinental Exchange and Frankfurt-based Eurex AG.

The five generic types of swaps, in order of their quantitative importance, are: interest rate swaps, currency swaps, credit swaps, commodity swaps and equity swaps. There are also many other types of swaps.

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[1] https://en.wikipedia.org/wiki/Swap_(finance)

- ► *Interest rate swaps*.
- ► *Currency swaps*. A currency swap involves exchanging principal and fixed rate interest payments on a loan in one currency for principal and fixed rate interest payments on an equal loan in another currency;
- ► *Commodity swap*. A commodity swap is an agreement whereby a floating (or market or spot) price is exchanged for a fixed price over a specified period. The vast majority of commodity swaps involve crude oil;
- ► *Total return swap*. A total return swap is a swap in which party A pays the total return of an asset, and party B makes periodic interest payments. The total return is the capital gain or loss, plus any interest or dividend payments. Note that if the total return is negative, then party A receives this amount from party B. The parties have exposure to the return of the underlying stock or index, without having to hold the underlying assets. The profit or loss of party B is the same for him as actually owning the underlying asset;
- ► *Swaption*. An option on a swap is called a swaption. These provide one party with the right but not the obligation at a future time to enter into a swap.

1.2 Terminology

In this section we provide a list of terms used throughout this guide.

- ► **Libor rate**. The London Inter-bank Offered Rate is an interest-rate average calculated from estimates submitted by the leading banks in London. Each bank estimates what it would be charged were it to borrow from other banks. The resulting rate is usually abbreviated to Libor or LIBOR, or more officially to ICE LIBOR (for Intercontinental Exchange Libor). It was formerly known as BBA Libor (for British Bankers' Association Libor or the trademark bba libor) before the responsibility for the administration was transferred to Intercontinental Exchange. It is the primary benchmark, along with the Euribor, for short-term interest rates around the world.
- ► Bankers' acceptance (**BA**) rate. A banker's acceptance is a promised future payment, or time draft, which is accepted and guaranteed by a bank and drawn on a deposit at the bank. The banker's acceptance specifies the amount of money, the date, and the person to whom the payment is due. After acceptance, the draft becomes an unconditional liability of the bank. But the holder of the draft can sell (exchange) it for cash at a discount to a buyer who is willing to wait until the maturity date for the funds in the deposit. The BA rate in the legal agreements usually refers to Libor, CDOR, or another base rate.
- ► **Prime rate**. The prime lending rate, is the annual interest rate major banks and financial institutions use to set interest rates for variable loans and lines of credit, including variable-rate mortgages. In the US the Prime rate is typically set at 3% premium relative to Federal Funds rate.
- ► **Federal Funds rate.**[2] The federal funds rate is the interest rate at which depository institutions in the US (banks and credit unions) lend reserve balances to other depository institutions overnight on an uncollateralized basis. The federal funds **effective** rate is the weighted average of rate across all such transactions. The federal funds **target** rate is determined by a meeting of the members of the Federal Open Market Committee which normally occurs eight times a year about seven weeks apart. The committee may also hold additional meetings and implement target rate changes outside of its normal schedule. The Federal Reserve (**Feds**) uses open market operations to make the federal funds effective rate follow the federal funds target rate.

[2] https://en.wikipedia.org/wiki/Federal_funds_rate.

► **SOFR rate**. is a secured interbank overnight interest rate and reference rate established as an alternative to LIBOR, which is published in a number of currencies and underpins financial contracts all over the world. Because LIBOR is derived from banks' daily quotes of borrowing costs, banks were able to manipulate the rates through lying in the surveys. Deeming it prone to manipulation, UK regulators decided to discontinue LIBOR.[3]

There are multiple types of SOFR rates.[4] Specifically,

1. Daily simple SOFR. It is calculated using simple interest over the interest period and therefore is not known in advance.

2. SOFR compounded in arrears. Under the interest calculation approach, simple SOFR daily rate is compounded over the interest period (and therefore is not known in advance). The compound rate is calculated using the following equation:

(1.1)
$$i^{sofr} = \left(\frac{sofr\ index_{end}}{sofr\ index_{start}} - 1\right) \times \frac{360}{d}$$

where $sofr\ index_{start}$ is SOFR index[5] at the beginning of the period, $sofr\ index_{end}$ is SOFR index at the end of the period, and d is the number of days in the period.

Assuming that SOFR index in period t is estimated as $sofr\ index_t = \left(1 + \frac{i^{sofr,daily}}{360}\right)^t$ (where t is the number of days over which the daily SOFR is compounded[6]), the above equation can be represented equivalently as follows:

(1.2)
$$1 + i^{sofr} \times \frac{d}{360} = \left(1 + \frac{i^{sofr,daily}}{360}\right)^d = \frac{sofr\ index_{t+d}}{sofr\ index_t}$$

3. SOFR compounded in advance. Under the approach, simple SOFR daily rate is compounded over a historical period of 30, 90, or 180 days (and therefore is known in advance); and

4. Forward-looking term SOFR (Term SOFR). The ARRC[7]-recommended Term SOFR has not yet been developed. On 21 April 2021, the CME Group, which is a leading derivatives market operator, announced that it had developed Term SOFR reference rates (**CME Term SOFR**) calculated using overnight SOFR rates together with market expectations implied from derivatives markets.[8] On Bloomberg, Term SOFR is reported via TSFR series,

► **CORRA.** Canadian Overnight Repo Rate Average – measures the cost of overnight general collateral funding in Canadian dollars using Government of Canada treasury bills and bonds as collateral for repurchase transactions. The Bank of Canada provides CORRA as a public good, at no cost to users and data distributors.

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[3] https://en.wikipedia.org/wiki/SOFR.

[4] https://mcmillan.ca/insights/sofr-fundamentals-what-we-know-so-far/

[5] SOFR index is published on SOFR administrator website: https://www.newyorkfed.org/markets/reference-rates/sofr-averages-and-index.

[6] In the equation, daily SOFR rate is assumed to be constant. In a generic case, the power function is replaced with a cumulative product function.

[7] Alternative Reference Rates Committee, https://www.newyorkfed.org/arrc.

[8] In loan agreements the reference to Term SOFR is described as follows: "Term SOFR Screen Rate means the forward-looking SOFR term rate administered by CME (or any successor administrator satisfactory to the Administrative Agent) and published on the applicable Reuters screen page (or such other commercially available source providing such quotations as may be designated by the Administrative Agent from time to time".

The CORRA Compounded Index is a measure of the cumulative impact of CORRA compounding over time, starting from a base value of 100 on June 12, 2020. The index can be used to calculate CORRA compounded between any two dates.[9]

- **Base rate**. Base rate refers to the floating interest rate index, which is used as a base to calculate floating interest rate on the loan using the following equation.

$$i = i_{base} + \pi \tag{1.3}$$

where i_{base} is the base rate and π is the applicable margin.[10] Loan legal agreements typically include the definitions of the base rate used for interest calculations and provide a schedule of applicable margins, which may depend on certain financial covenants (such as for example Debt/EBITDA). Most common base rates are USD Libor, CAD CDOR, EUR Euribor, and other.

Loan agreement often include several alternative base rates. A loan agreement may specify the base rate depending on the currency in which loan advances are drawn or provide an option to draw advances either using BA rate (e.g. Libor) or Prime rate. In most cases the interest rate calculated using Prime rate is higher than interest based on the BA rate.

- **Basis swap**. A basis swap is an interest rate swap which involves the exchange of two floating rate financial instruments. A basis swap functions as a floating-to-floating interest rate swap under which the floating rate payments are referenced to different bases. The bases on the two legs of the swap may represent different frequencies of the same base rate (for example a 3-month Libor is swapped into a 6-month Libor) or different currencies (for example USD Libor rate is swapped into CAD CDOR rate).
- **Basis swap curve**. A basis swap curve maps a floating base rate from leg 1 of the swap into an equivalent floating base rate from leg 2 of the swap plus a swap premium / discount. The two legs of the basis swap typically have different currencies. Examples of Bloomberg basis swap curves are Appendix A.4.
- **Floating-to-fixed swap**. A floating-for-fixed swap is a contractual arrangement between two parties in which one party swaps the interest cash flows of fixed rate loan(s), with those of floating rate loan(s) held by another party. The principal of the underlying loans is not exchanged.
- **Floating-to-fixed swap curve**. A floating-to-fixed swap curve maps the floating base rate from leg 1 of the swap into an equivalent fixed rate on leg 2 of the swap. Both legs of the floating-to-fixed swap typically have the same currency. Examples of Bloomberg floating-to-fixed swap curves are Appendix A.4
- **Day count basis**. The day count basis quantifies periods of time to calculate the interest expense amount payable on each loan's interest payment date. Standard day count basis includes the following conventions: 30/360, Actual/360, Actual/Actual, Actual/365.
- **FX spot rate**. FX spot rate is the current exchange rate at which a currency pair is traded.
- **FX forward rate**. The forward exchange rate (also referred to as forward rate or forward price) is the exchange rate at which a bank agrees to exchange one currency for another at a **future** date when it enters into a forward contract with an investor. FX spot and forward rates are used as a basis to price future cash flows denominated in different currencies.

[9] https://www.bankofcanada.ca/rates/interest-rates/corra/.

[10] In some cases, a loan agreement may include a floor on the base rate. For example, the base rate may be calculated as a maximum of the floor (e.g., 1%) and the interest rate index (e.g., Libor).

Section 2 Cross-Currency Fixed-to-Fixed Swaps

This section presents the estimation of a cross-currency fixed-rate swap using the arbitrage pricing approach. The approach is based on the NPV valuation of two otherwise identical bond instruments denominated in different currencies. The two cash flows correspond to two legs of a swap instrument.

The cross-currency fixed rate swap analysis is presented in three steps:

1. First, in Section 2.1, a proxy equation (2.1) is derived based on a simple proxy arbitrage argument.
2. Next, in Section 2.2, the equation (2.1) is replaced with equation (2.5), which is an exact equation for the cross-currency fixed-to-fixed swap rates. The discount rate in the equations is estimated to match the risk profile of swapped rates.
3. Finally, in Section 2.4, equation (2.5) is derived for a special case of constant model parameters (including constant risk-free rate, risk-premium, and FX forward rate).

2.1 'Simple' arbitrage model

'Simple' model is based on a famous **interest rate parity** equation[11] which relates the spread between foreign and domestic interest rates to the currency forward rates.

We use the following notation and terms throughout this guide. We refer to the currency on the first leg of the cross-currency swap as the local (domestic) currency and use notation A or D for the currency. In the swap tool the currency is referred to as the "from currency". We refer to the currency on the second leg of the cross-currency swap as the foreign currency and use notation B or F for the currency. In the swap tool the currency is referred to as the "to currency". The forward rates are assumed to be quoted from local to foreign currencies and denoted as $f^{A,B}$. In actual applications the default option is to use USD as the local currency ("from currency") and use the non-USD currency as the foreign currency ("to currency").

In a fixed-to-fixed cross-currency swap, coupon and principal payments of a bond in currency A into coupon and principal payments of a bond denominated in currency B. a high-level approach to estimate the cross-currency swap is based on the interest rate parity equation:

(2.1)
$$i_t^B \approx i_t^A + f_t$$

where i_t^D is the interest rate in domestic currency (USD), i_t^F is the interest rate in foreign currency (CAD), and

(2.2)
$$f_t = \frac{F_t}{S} - 1$$

is the forward rate between domestic and forward currencies (USD-to-CAD). The interest rate parity can be viewed as a one-period approximation of the interest rate swap. The interest rate parity is based on the following arbitrage argument. Suppose that two debt transactions are traded in the market that have all

[11] https://en.wikipedia.org/wiki/Interest_rate_parity

term equal with exception of the debt currency. Both debt transactions mature in the next period. We also assume that debt is risk free. Then investor has two alternative strategies.

- **Strategy A**: invest in the debt traded in domestic currency. The return from the investment strategy is $1 + i_t^D \times \Delta_t$, where Δ_t is the day count;
- **Strategy B**: invest in the debt traded in domestic currency and hedge all currency risk. The return from the investment strategy is $(1 + i_t^F \times \Delta_t) \times \frac{S}{F_t}$.

The two strategies must produce the same return. The condition is described by the following equation.

$$1 + i_t^B \times \Delta_t = (1 + i_t^A \times \Delta_t) \times (1 + f_t \times dt) \approx 1 + i_t^A \times \Delta_t + f_t \times dt$$

In practice, the maturity of the cross-currency swap exceeds one period. The simple equation above must be replaced then by a multi-period cash flow model and the swap rate is derived from the cash flow model. Conceptually however the cash flow model implements the same two investment strategies: (i) invest in debt denominated in domestic currency and (ii) invest in comparable debt denominated in foreign currency and hedge all FX risk with FX forward contracts. The approach is described in more detail in the sections below.

To estimate the payments denominated in another currency, cross-currency (FX) forward rates must be obtained or estimated. In the sections below we discuss three alternative approaches when the FX rates are obtained directly from Bloomberg or Reuters databases or alternatively implied FX rates are estimated from (i) swap curves reported by Bloomberg or Reuters or (ii) based on the government debt denominated in two currencies.

2.2 NPV model

In this section we derive the equations for a cross-currency swap model, discuss how they are applied to implement efficiently a cross-currency swap valuation tool, and present and example with the swap model cash flows that is generated by the tool. This is the NPV model implemented by Bloomberg as part of SWPM swap valuation tool (SWPM tool is discussed in detail in Appendix A).

2.2.1 NPV model equations

Suppose that $F_t^{A,B}$ is the FX forward rate from currency A to currency B and $S^{A,B}$ is FX spot rate. Suppose also that c^A are coupon payments denominated in currency A and principal amount is normalized to one. The principal can be converted into currency B, $X^B = 1 \times S^{A,B}$, and coupon and principal payments can be converted then back into currency A using forward rates. The NPV of the two cash flows must be equal to each other.

In the case of a single period, the equation is described as follows.

$$D_T \times (1 + c^A \times \Delta_T) = D_T \times (1 + c^B \times \Delta_T) \frac{S^{A,B}}{F_T^{A,B}}$$

or

$$c^B = \frac{1}{\Delta_T} \times \left(\frac{F_T^{A,B}}{S^{A,B}} - 1 \right) + c^A \times \frac{F_T^{A,B}}{S^{A,B}} \tag{2.3}$$

Equation (2.3) is applied for maturities $T < 1$.

In general case,

$$c^B = a_{0,T} + a_{1,T} \times c^A \tag{2.4}$$

For a multi-period model, the intercept and slope coefficients ($a_{0,T}$ and $a_{1,T}$) of the linear relationship (2.4) are derived using the equation described below.

$$\sum_{t=1,\dots,T-1} D_t \times c^A \times \Delta_t + D_T \times (1 + c^A \times \Delta_T) = \sum_{t=1,\dots,T-1} D_t \times \frac{c^B \times \Delta_t \times S^{A,B}}{F_t^{A,B}} + D_T \times (1 + c^B \times \Delta_T) \frac{S^{A,B}}{F_T^{A,B}}$$

After rearranging the terms, the swapped coupon rate c^B denominated in currency B can be estimated using the following linear transformation of coupon payment c^A denominated in currency A.

$$c^B = \frac{1}{\left[\sum_{t=1,\dots,T} D_t \times \frac{S^{A,B}}{F_t^{A,B}} \times \Delta_t \right]} \times \left(D_T \times \left(1 - \frac{S^{A,B}}{F_T^{A,B}} \right) + c^A \times \sum_{t=1,\dots,T} D_t \times \Delta_t \right) = a_{0,T} + c^A \times a_{1,T} \tag{2.5}$$

where

$$a_{0,T} = \frac{1}{\left[\sum_{t=1,\dots,T} D_t \times \frac{S^{A,B}}{F_t^{A,B}} \times \Delta_t \right]} \times D_T \times \left(1 - \frac{S^{A,B}}{F_T^{A,B}} \right) \tag{2.6}$$

is an intercept and

$$a_{1,T} = \frac{1}{\left[\sum_{t=1,\dots,T} D_t \times \frac{S^{A,B}}{F_t^{A,B}} \times \Delta_t \right]} \times \sum_{t=1,\dots,T} D_t \times \Delta_t \tag{2.7}$$

The linear relationship is estimated sequentially each maturity term $T \geq 1$.

Equations (2.5) through (2.7) describe a linear relationship between the interest rates denominated in different currencies. The linear relationship is estimated for each date and each maturity term based on respective forward and discount rates and applied then to calculate the swap for each interest rate c^A.

2.2.2 NPV model cash flows

The derived above equations present the relationship between bond coupons rates denominated in different currencies. The swap calculations can also be illustrated using the cash flows estimated for both legs of the swap instrument. The cash flows are presented by the following exhibit.

Exhibit 2.1 FX swap cash flow model

Period	Discounts	Forward rates	Cash Flow: Leg A	Cash Flow: Leg B	Cash Flow PV
$t = 0$	$D_0 = 1$	$F_0^{A,B} = S^{A,B}$	0	0	0
…	…	…	…	…	…
t	D_t	$F_t^{A,B}$	$c^A \times \Delta_t$	$c^B \times \frac{S^{A,B}}{F_t^{A,B}} \times \Delta_t$	$D_t \times \left(c^A - c^B \times \frac{S^{A,B}}{F_t^{A,B}}\right) \times \Delta_t$
…	…	…	…	…	…
$t = T$	D_T	$F_T^{A,B}$	$1 + c^A \times \Delta_T$	$(1 + c^B \times \Delta_T) \times \frac{S^{A,B}}{F_T^{A,B}}$	$D_T \times \left[(1 + c^A \times \Delta_T) - (1 + c^B \times \Delta_T) \times \frac{S^{A,B}}{F_T^{A,B}}\right]$

Under the NPV valuation, the sum of the cash flows PV values in the last column of the cash flow table must be equal to zero. The property can be used as a **validation test** of the NPV valuation results.

The generated cash flows in the NPV model can also be used to validate the results of the swap valuation against the Bloomberg SWPM tool. The SWPM tool produced a detailed information including the discount curves and the swap model cash flows, which are discussed in detail in Appendix A.

2.3 FX forward rates

The FX forward rates are key inputs for the cross-currency swap valuation. The FX rates can be estimated directly from the forward curves or implied FX rates can be estimated as discussed below. In terms of implementation simplicity, the three approaches can be ranked as follows.

1. Bloomberg FX forward series. This is the simplest approach as it is based on directly available Bloomberg estimates of the FX forward curves;
2. Implied FX forward rates based on swap curves. This is a more complex approach as it requires to estimate the implied FX rates based on the term structures of interest rates denominated in domestic and foreign currencies. The term structures are estimated directly based on the swap curves.
3. Implied FX forward rates based on sovereign debt issuances. This is the most complex approach among the three approaches as it requires to estimate both (i) the term structures of domestic and foreign currency denominated interest rates based on searches for sovereign debt, and (ii) estimate the implied FX forward rates based on the constructed term structures of interest rates.

2.3.1 Bloomberg FX Forward Series

This is a direct approach to estimate the FX forward rates. Bloomberg reports FX forward curves for the majority of the currencies. For example, USD-to-CAD FX spot and forward rates with different maturities are reported using CAD, CAD1, CAD2, …, where CAD is the USD-to-CAD FX spot rate curve, CAD1 is the USD-to-CAD 1-year FX forward rate curve, etc.

2.3.2 Implied FX forward rates based on swap curves

For many currencies Bloomberg SWPM estimates implied FX rates based on the respective swap curves [**NTD: Need to check**]. The approach uses the following swap curves as inputs (for simplicity we describe the approach for the USD and CAD currency pair):

- Fixed-to-float swap curves with different maturities in domestic currency (USD). The swap values are denoted as $c_t^{A,float-to-fixed}$;
- Basis swap curves with different maturities from domestic to foreign currency (CAD-to-USD)[12]. The swap values are denoted as $c_t^{B,A,float-to-float}$;
- Fixed-to-float swap curves with different maturities in foreign currency (CAD). The swap values are denoted as $c_t^{B,float-to-fixed}$

The three curves allow to estimate a mapping from a specific term structure of interest rates denominated in the domestic currency (USD) into the term structure of interest rates denominated in the foreign currency (CAD). The mapping is estimated a follows.

- The term structure of fixed interest rates in foreign currency is estimated as $c_t^B = c_t^{B,float-to-fixed}$;
- The term structure of fixed interest rates in foreign currency is estimated as $c_t^A = c_t^{B,A,float-to-float} + c_t^{A,float-to-fixed}$.

The implied FX forward rate is estimated using the equation (2.5) based on the term structures c^A and c^B. The implied FX forward rates are described by the following equations (derived in Appendix F).

(2.8) $$f_t^{A,B} = \frac{D_t \times (1 + c_t^B \times \Delta_t)}{D_t \times (1 + c_t^A \times \Delta_t) + A_{t-1}}$$

where

(2.9) $$A_t = \sum_{s=1,\dots,t} D_s \times \left(c_t^A - \frac{c_t^B}{f_s^{A,B}}\right) \times \Delta_t = A_{t-1} + D_t \times \left(c_t^A - \frac{c_t^B}{f_t^{A,B}}\right) \times \Delta_t$$

2.3.3 Implied FX forward rates based on government debt

The approach is similar to the swap curve-based approach. The key difference is that the term structures of the interest rates denominated in the domestic and foreign currencies are estimated based on the searches for government debt denominated in the two currencies. The approach is summarized as follows.

- Search for the sovereign debt transactions denominated in both local and USD currencies;
- Estimate term structures of interest rates for both local and USD currencies (estimation of the interest rate term structure is discussed in the respective "Interest Rate Benchmarking Analysis" guide;
- Estimate the implied FX forward rates using equations (2.8) and (2.9).

[12] The basis swap curves on Bloomberg typically have non-USD currency on the first leg and USD currency on the second leg.

The approach is typically applied when neither FX forward curves nor swap curves are available.

2.4 NPV model with constant parameters

This section derives the equivalent of equations (2.5) – (2.7) for the fixed interest rates which also include a risk premium.

2.5 Summary of Cross-Currency Swap Valuation

The valuation of a cross-currency swap can be summarized as follows.

1. **FX forward rates**. Estimate the FX forward rates using one of the three alternative approaches:
   - ► Obtain the FX forward rates directly from Bloomberg database;
   - ► Estimate implied FX forward rates based on Bloomberg swap curves;
   - ► Estimate implied FX rates based on the search of related government debt denominated in the two currencies;
2. **Swap equations**. Estimate the interest rate swap using the equation

   (2.10) $$c^B = a_{0,T} + c^A \times a_{1,T}$$

   where $a_{0,T}$ and $a_{1,T}$ are described respectively by equations (2.6) and (2.7). Note that equations (2.6) and (2.7) can be estimated not only for a single date but using a time series of FX forward rates. As a result, the above equation can be applied to estimate a series of interest rate swaps.

   The swap equation is linear in the yield rates c^A and c^B. Therefore, the reverse swap can be estimated directly from the above equation as

   (2.11) $$c^A = \frac{-a_{0,T} + c^B}{a_{1,T}}$$

3. **Equation implementation**. The swap estimation is implemented in the ac.Swap tool which is described in detail in Appendix B. The tool implements swap calculations as follows.
   - ► **Set swap calculator**. At step one, the swap tool calculates $a_{0,T}$ and $a_{1,T}$ coefficients. Note that the same coefficients can be applied to both direct and reverse swap. Therefore, it does not matter which currency to use at each leg. By default, it is recommended to use USD on leg one of the swap and the other currency set on leg two of the swap. The curves must be consistent with the currencies on the two legs of the swap. For example, if USD and CAD currencies are used in the first and second legs of the swap, then USD-to-CAD forward curve must be estimated, and discount rates must be estimated based on the USD yields. (To estimate implied FX forward rates, the respective parameters of the FX calculator must be set as (i) USD floating-to-fixed curve and (ii) USD-to-CAD floating-to-fixed curve calculated as USD-to-CAD basis swap curve plus CAD floating-to-fixed curve);
   - ► **Swap yield series**. After the $a_{0,T}$ and $a_{1,T}$ coefficients are set in the swap calculator, it can be applied to an arbitrary yield curve. The calculator uses the currency of the series to identify whether a direct or reverse swap is estimated and apply respectively equation (2.10) or (2.11). The calculator can be applied consecutively to multiple yield series with different maturities.
4. **Cash Flows** (optional): construct the cash flows of the swap model for a specific date.

5. **Validation**. Validate the results of the swap valuation by performing internal validation and validation against the Bloomberg SWPM tool (see Appendix F for validation analysis details).

Section 3 Fixed-to-Floating Interest Rate Swap

This section discusses the interest rate swap models for the fixed-to-floating swaps.

3.1 Swaps with standard base rate

In the case of a standard base, such as Libor, SOFR, CDOR, etc., there are two alternative options to estimate the fixed-to-float interest rate swap:

1. Perform interest rate swap using Bloomberg or Eikon interest rate swap tools. The approach is discussed in detail in Appendix A and Appendix C. The approach is based on exact NPV valuation of swap cash flows and is similar to the approach discussed in Section 2. A disadvantage of the approach is that it requires manual estimation for each interest rate swap.
2. Perform interest rate swap using respective swap curves estimated by Bloomberg and Eikon. The description of the interest rate swaps is provided in Appendix B and Appendix D. A short description of the approach is discussed in this section below. The technical details of swap curve estimation is discussed in Appendix H.

Floating-to-fixed rate swaps are typically straightforward to estimate since a floating-to-fixed swap curve is available through Bloomberg. Suppose that we need to convert the floating rate $\mathrm{i}_{\mathrm{base}}+\pi$ into an equivalent fixed rate denominated in the same currency. Then we perform swap valuation by following the steps below.

- ► Identify a Bloomberg floating-to-fixed swap curve for the related base rate;
- ► Apply the swap curve to convert the base rate i_{base} into equivalent fixed rate $\mathrm{i}_{\mathrm{fixed}}^{\mathrm{swap}}$;
- ► Estimate the fixed rate equivalent as

  (3.1) $$i^{fixed} = i^{swap} + \pi$$

  where i^{swap} is the swapped rate and π is the applicable margin. The equation is derived directly based on the definition of the swap rate as a fixed-rate equivalent of the base rate. Application of the equation is conditional on the existence of the swap curve estimated and published by Bloomberg or other reputable sources. (Methodology for the swap curve estimation is discussed in Appendix H.2).
- ► Perform additional swap adjustments for differences in interest payment frequency or day count basis, if the frequency and day count basis of the Bloomberg swap curve are different from the frequencies and day count basis applied in the floating-to-fixed swap.

3.2 Swaps with non-standard base rate

A base of a floating rate is non-standard, if it cannot be selected in the respective Bloomberg or Eikon interest rate swap tool or if respective swap curves are not estimated by Bloomberg or Reuters. In this case, a proxy standard rate is selected for the non-standard rate. The swap is performed then using a standard base and standard base is replaced with a non-standard base (estimated via linear relationship).

Linear swap curve relationship

Specifically, the interest rate swap estimation can be summarized by the following equation:

(3.2) $$i^{fixed} \sim i^{base,\ std} + \pi = \alpha + \beta \times i^{base,\ non-std} + \pi$$

where premium π is estimated as the difference between the fixed rate and the swap rate (i^{swap}) for the proxy standard base rate:

(3.3) $$\pi = i^{fixed} - i^{swap,std}$$

The linear relationship between the non-standard and standard base rates is described by the following equation (estimated using simple regression analysis)

(3.4) $$i^{base,\ std} = \alpha + \beta \times i^{base,\ non-std}$$

Equations (3.2) and (3.4) are used to convert the floating rate margin π into the fixed rate i^{fixed}.

If, on the other hand, a fixed rate is converted to a floating rate using the following equation (derived directly from equation (3.2)):

(3.5) $$\frac{i^{fixed} - \alpha}{\beta} = i^{base,\ non-std} + \frac{\pi}{\beta}$$

so that the swap curve for a non-standard base is described by the following equation

(3.6) $$i^{swap,non-std} = \frac{i^{swap,std} - \alpha}{\beta}$$

and the premium is estimated as described in equation (3.3)

(3.7) $$\pi = i^{fixed} - i^{swap,non-std}$$

Simple swap curve relationship

In a simple case, the slope coefficient must be **set to one** ($\beta = 1$), and the intercept parameter is estimated using the following equation:

(3.8) $$\alpha = avg[i^{base,\ std} - i^{base,\ non-std}]$$

The average statistics in the above equation can be replaced by an alternative statistics (e.g., median which is more robust to outliers). The swap curve relationship is presented then as follows.

(3.9) $$i^{swap,non-std} = i^{swap,std} - \alpha$$

3.2.1 Selection of proxy base rates

Swap to a non-standard base depends on the selection of a standard base proxy. There are two options to select proxy rates:

1. A proxy standard base rate is selected to match the currency and risk profile of the non-standard base rate. For example, 3M C$ CDOR rate is selected as a proxy for the 3M C$ government T-bill rate.
2. A proxy standard base rate is selected to match the same rate but quoted in different currency. For example, US$ Prime rate can be selected as a proxy to the C$ Prime rate. The approach should be considered when a reliable swap adjustment can be performed for the rate in one currency but not in the other currency. The additional support to the approach is based on the fact that third-party loan agreements typically report the same margin over base rates reported in different currencies.[13]

Examples with proxy rates selection are presented below.

3.2.2 Prime rate swaps

Prime rate for US$ and C$ currencies are available via the Bloomberg swap tool. However, the Prime rate for C$ currency did not work properly (SWPM tool was effectively performing swap from CDOR rate when the Prime rate was selected for the C$ currency). For the US$ currency the Bloomberg's swap from the Prime rate seem to function correctly. However, since the direct swap curve for the Prime rate was not available, we include the discussion of the US$ Prime rate swap in this section too.

USD Prime swaps

CAD Prime swaps

3.2.3 T-Bill rate swaps

Certain interest rates are defined as a spread over government short-term bond rates. For example, Canadian PLOI rate[14] is defined as 4% + regular prescribed rate and regular prescribed rate is defined as a lagged average three-month Treasury bill rate.

Bloomberg or Eikon interest rate swap tools do not allowing selecting Canadian 3-months T-Bill rate as a base rate. Therefore, an approximate interest rate swap is performed by replacing the 3-months T-Bill rate with a close proxy which can be either selected in the swap tool or which has a swap curve estimated and reported by Bloomberg or Eikon.

In the example of the PLOI rate, the equation (3.4) is estimated respectively as follows.

$$i^{GCAN-3M} = \alpha + \beta \times i^{CDOR-3M} \quad (3.10)$$

where GCAN3M is Bloomberg series for the Canadian 3-Months T-Bill and CDOR3M is Bloomberg series for the Canadian CDOR rate, which can be converted into a fixed-rate equivalent using Bloomberg CDSW swap curve.

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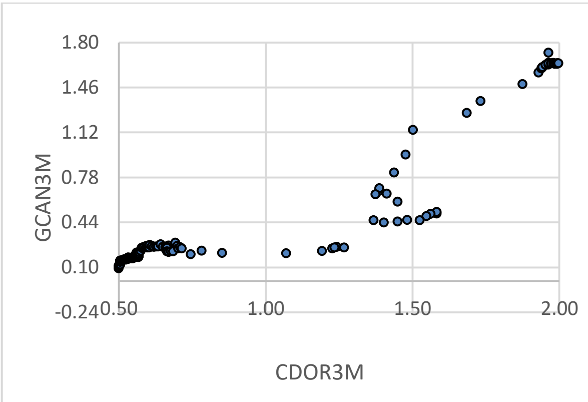
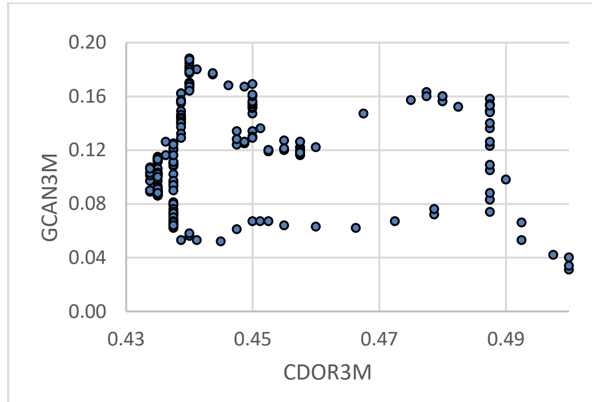
[13] It is typically assumed that FX impact is accounted via the movements in the base rates and the margin over the base rate (which measures credit risk and does not depend directly on the loan currency) is fixed.

[14] Pertinent loan or indebtedness, https://www.canada.ca/en/revenue-agency/services/tax/prescribed-interest-rates.html.

To estimate robustly the linear relationship (3.10), the range of historical values should be selected to ensure that there is a significant variation in the rates over the range. Otherwise, small variation in the rates may produce non-reliable and unintuitive linear relation estimates. An example is illustrated in the exhibit below.

Exhibit 3.1 Linear relationship between GCAN3M and CDOR3M series

FY2020 period	FY2021 period
	

The FY2020 data shows a high variation in both series and a strong linear relationship between them. The FY2021 series shows a very low variation in the two series and the break-down in the linear relationship.

The swap from the GCAN3M base rate to an equivalent C$ fixed rate is described by the following equation.

$$i^{GCAN-3M} + \pi = \alpha + \beta \times i^{CDOR-3M} + \pi = \alpha + \beta \times i^{swap} + \pi = i^{fixed} \quad (3.11)$$

The fixed-rate equivalent can be used to test the arm's length nature of the floating rate equal to GCAN3M rate plus margin ($i^{GCAN-3M} + \pi$).

Section 4 Swap Adjustment for Interest Payment Terms Differences

In addition to major differences between interest rates (such as currency and fixed/float type), the interest rates also differ in interest payment frequency and day count basis. Typically, these adjustments are relatively small (a few basis points). The formulas below derive approximate adjustments for the terms' differences.

4.1 Adjustment for Frequency

With higher frequency of interest payments, the bond holder has an option to reinvest the interest income and generate a higher return. Therefore, an adjustment for interest payment frequency need to be performed generally. Let's illustrate it by example.

Suppose that interest rate on a bond with semi-annual interest payments is i (and principal amount is normalized to 1). Suppose that the bond terms are amended to annual frequency payment.

With semi-annual payments, the bond holder receives $\frac{i\%}{2}$ interest amount, which is reinvested then into a new asset for a six-month period. Suppose that the short-term reinvestment rate is i^*. Then additional return on the investment is $\Delta i = \frac{i \times i^*}{4}$.

For a general frequency parameter $f = 2, 3, 4, 6, 12$, the equation that estimates the additional benefit to the lender from a higher frequency interest payment is described by the following equation:

(3.12) $$\Delta i(f) = \sum_{\mathrm{k}=1}^{f-1} \frac{i}{f} \times \left(\left(1 + \frac{i^*}{f}\right)^{f-\mathrm{k}} - 1 \right) = \frac{i}{f} \times \left(1 + \frac{i^*}{f}\right) \times \left[\frac{\left(1 + \frac{i^*}{f}\right)^{f-1} - 1}{\frac{i^*}{f}} \right] - \frac{i}{f} \times (f-1)$$

Since the higher interest payment frequency is the benefit to the lender, the interest rate needs to be adjusted by the respective discount. If the adjustment is performed from frequency f_A to frequency f_B, then the discount adjustment is calculated as follows:

(3.13) $$\Delta i(f_B, f_A) = \Delta i(f_B) - \Delta i(f_A)$$

In general, the refinancing rate i^* needs to be calculated separately. For simplicity, we use a one-year risk-free rate (estimated from the discount rates used in swap calculations) as a proxy for the refinancing rate.

The equation (3.12) can be approximated as follows:

(3.14) $$\Delta i(f) = \frac{i}{f} \times \frac{i^*}{f} \sum_{k=1}^{f-1} (f-k) = i \times \left(i^* \times \frac{f-1}{2f} \right)$$

Based on the equations (3.13) and (3.14), the discount adjustment for interest rate frequency is estimated as follows

$$\Delta i(f_B, f_A) = i \times i^* \times \left[\frac{f_B - 1}{2f_B} - \frac{f_A - 1}{2f_A}\right] \quad (3.15)$$

The multiplier

$$\alpha = 1 - i^* \times \left[\frac{f_B - 1}{2f_B} - \frac{f_A - 1}{2f_A}\right] \quad (3.16)$$

is applied to perform frequency swap adjustment.

$$c^B = \alpha \times c^A \quad (3.17)$$

The discount rate i^* is selected at the median tenor ($t = \frac{T}{2}$) value.

4.2 Adjustment for Day Count

There are typically four major day count conventions used in the interest calculations: 30/360, Act/Act, Act/365, and Act/360.

The 30/360, Act/Act, and Act/365-day count basis correspond to approximately the same number of interest payment days during the calendar year (Act/365-day count basis has one extra interest payment day during the leap years with 366 calendar days). Therefore, no adjustment is performed for the day count basis if both legs of the swap have one of the three above day count conventions.

Act/360-day count basis has approximately 5-6 additional interest payment days during the calendar year. The adjustment in this case is performed as follows:

$$y^{non-Act/360} = y^{Act/360} \times \frac{Act}{360} \sim y^{Act/360} \times \left(1 + \frac{5}{360}\right) \sim y^{Act/360} + y^{Act/360} \times 1.4\% \quad (3.18)$$

For example, if $y^{Act/360} = 5\%$, then adjustment from Act/360 to non-Act/360-day count basis is equal approximately to 5% x 1.4% = 7bps

Appendix A Bloomberg Swap Manager Tool (SWPM)

The Bloomberg interest rate swap tool (SWPM) is a standard tool applied in transfer pricing to estimate cross-currency and / or fixed-to-floating interest rate swaps. The output of the swap can be either (i) interest rate on leg one or leg two of the swap or (ii) swap NPV value (or respectively swap premium).

A.1 SWPM tool terminology

The SWPM tool uses the following parameters:[15]

- *Curve date*. Date of the curves (discount, FX, or swap) and volatility data used in the analysis.
- *Valuation date*. Settlement date used for swap valuation.
- *Effective date*. Start date of the swap contract.
- *Maturity date*. Termination date of the swap contract.
- *Premium*. Swap premium estimated as % of the swap notional amount.

The SWPM tool estimates implied FX rates and discount factors based on the respective swap curves. The tool reports the estimated discount factors and the cash flows for the two legs of the swap including the estimated implied FX rates. An example of Bloomberg SWPM tool input and output is described below.

A.2 SWPM tool output

A standard output of the SWPM tool is illustrated below for fixed-to-fixed and fixed-to-float interest rate swaps.

A.2.1 Cross-currency fixed-to-fixed swap

A Bloomberg print screen with the results of the SWPM cross-currency fixed-to-fixed swap output is illustrated in the exhibit below.

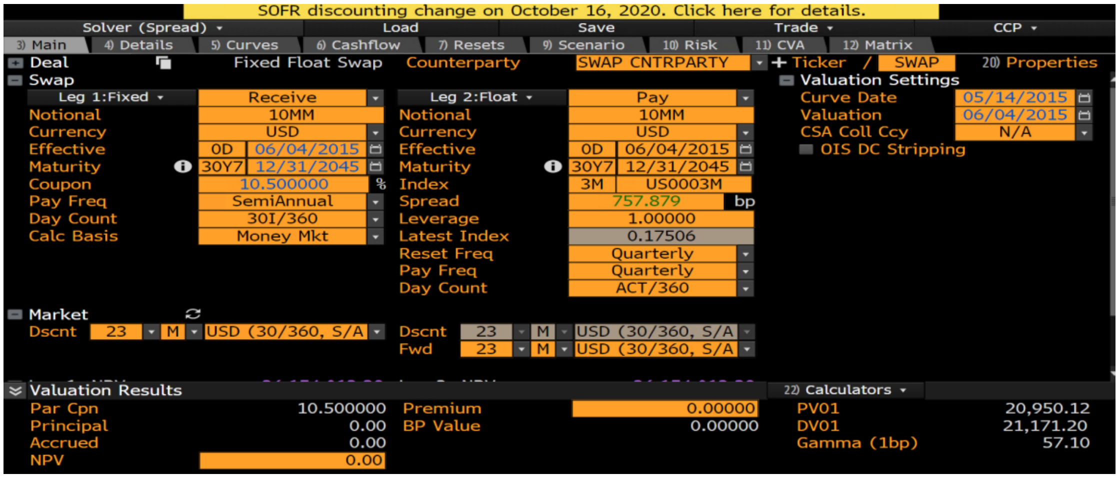
Exhibit A.1 SWPM cross-currency fixed-to-fixed swap output

A.2.2 Fixed-to-float swap

A Bloomberg print screen with the results of the SWPM fixed-to-float swap output is illustrated in the exhibit below.

[15] We provide Bloomberg's description of each field.

Exhibit A.2 SWPM fixed-to-float swap output



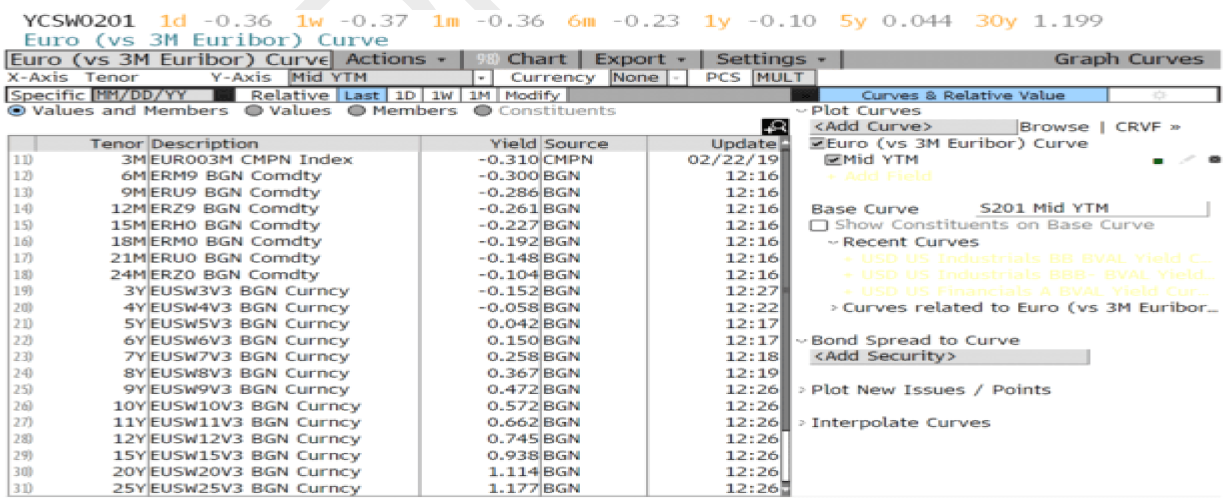
The base rate in the fixed-to-float swap can be set to a Libor (with arbitrary frequency) or to a Prime rate. The SWPM tool calculates the base rate forward rates (referred to as reset rates) and calculates the swap cash flows. The cash flows on the floating leg are calculated using the base rate reset rates.

The discount rates in the tool are calculated based on the Libor swap curves (USSW curve) [**Check**].

A.3 SWPM tool inputs

The SWPM tool fields are summarized in section A.1. In this section we describe the tool parameters used for discount and implied FX calculations. We discuss the parameters in the context of USD-to-EUR swap example. The USD-to-EUR swap valuation uses the following default curves as inputs.

- EUR floating-to-fixed curve: Euro (vs 3M Euribor) curve which is constructed fir different tenors using the following curves:



The curve uses Euribor series for 3-month maturity, Euribor futures for 6 month up to 2 years maturity, and swap curves for maturities of 3 years or higher.

- .

A.4 SWPM tool cash flows

The cash flow model in SWPM tool is the same as in the ac.Swap tool. The difference in the swap valuation output is attributable to the differences in the in the cash flow model inputs: (i) difference in discount factors and (ii) difference in the implied FX rates. Since discount factors are applied on both legs of the swap, the difference in the discount factors typically has a small impact on the results of swap valuation. Difference in the implied FX rates has a significantly larger impact on the results of swap valuation.

A.5 Base rate floor adjustment

In many loan agreements, the terms include a floor on the base rate. The base rate floor limits from below the interest rate on the loan and therefore should be adjusted via the reduction in the interest rate spread. The adjustment can potentially be material and equal to 20-30bps depending on current value of the base rate and its volatility.

There are two approaches discussed below to adjust for the base rate floor.

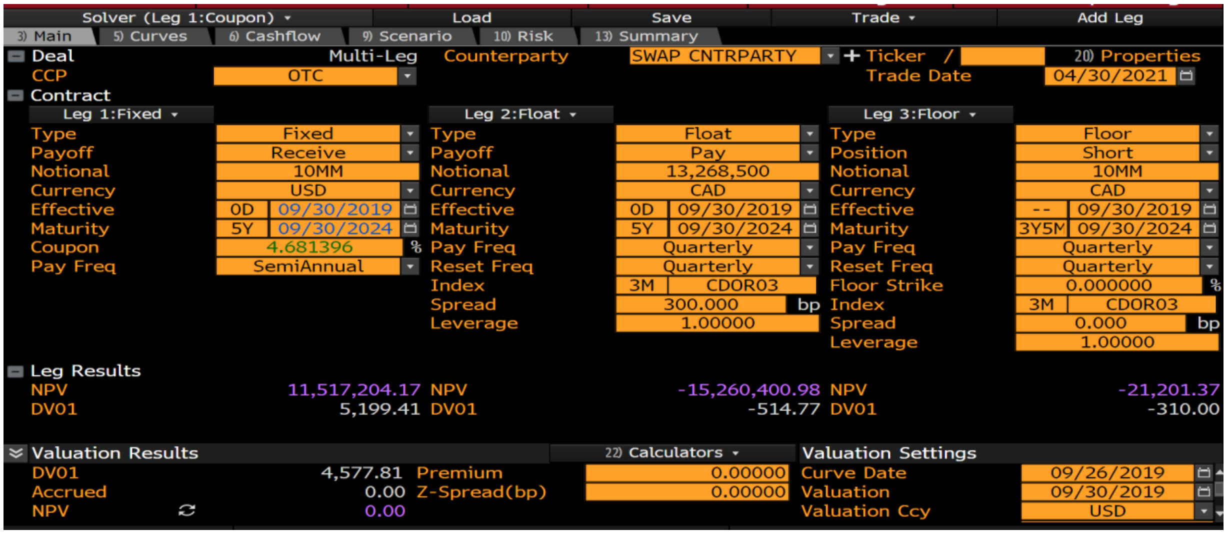
3. Under the first approach, the adjustment is performed directly using SWPM tool as discussed below. This is a recommended approach as it takes into account both current value of the base rate and its volatility.
4. Under the second approach, the base rate floor is imposed on the forward base rate values and spread adjustment is estimated so that the NPV of the swap cash flows with unadjusted base rate equals to the NPV of the cash flows with adjusted cash flows. The approach does not take into account base rate volatility and will produce zero adjustment if forward base rates are all above the base rate floor. The approach should be applied only as a corroborative approach if current or forward base rates are below the base rate floor. The estimated adjustment under the approach will be biased downward.

The two approaches are discussed in more detail below.

A.5.1 Adjustment using SWPM tool

Floor for the base rate are often included in the loan agreements. Adjustment for the base rate floor can be quite material: 30-50bps. The adjustment for the interest rate floor can be performed using Bloomberg's SWPM tool as illustrated below.

The float-to-fixed swap with the base rate floor constraint is performed as follows. Run the SWPM float-to-fixed swap tool. In the Leg2: Float menu select 'Add Leg' and select 'Floor' and 'USD' parameters. Select short position to limit the movement of the base rate below the threshold value. (Note: in cross-currency float-to-fixed swap, first currency needs to be selected and then the 3rd leg added. Otherwise the tool produces some meaningless results). The SWPM output is illustrated below.



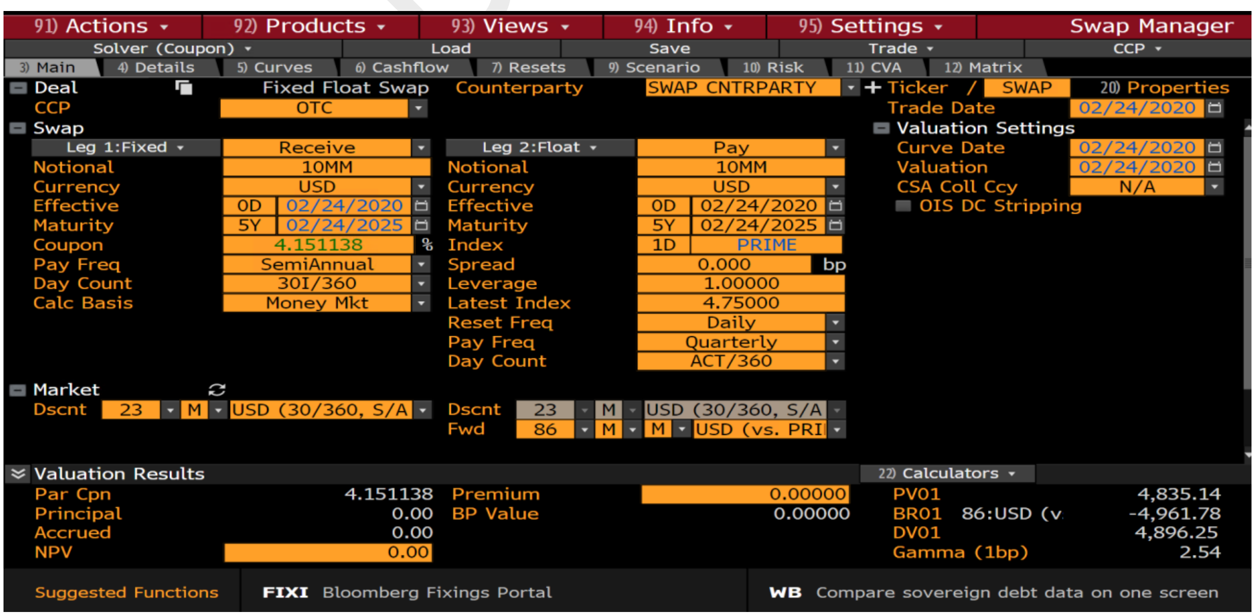
A.5.2 Adjustment using forward base rates

A.6 Swap to Prime rate

The relationship between Prime rate and the central bank policy rate is discussed in Appendix G.1. Therefore, the swap into the Prime rate must be consistent with the pattern. In this section, we show Bloomberg SWPM print screens for the Prime rate and test the consistency of the SWPM results with the historical patterns of Prime rate premium.

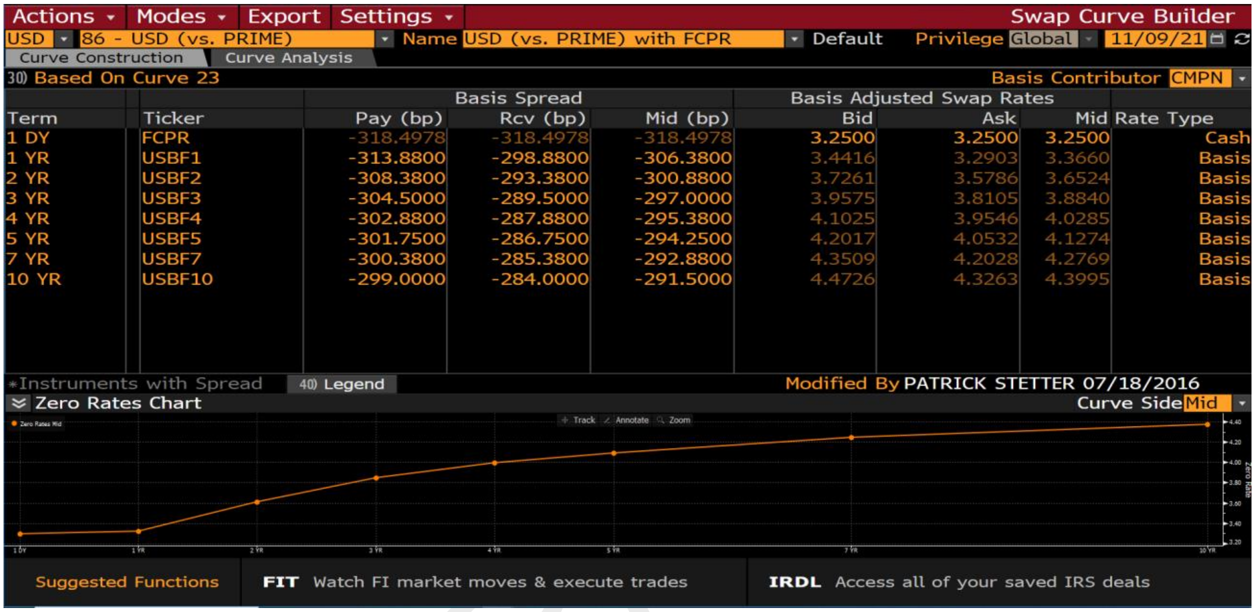
A.6.1 Swap tool input/output

The detailed inputs of the Prime to fixed swap tool are shown in the exhibit below.



The tool uses the following swap curves

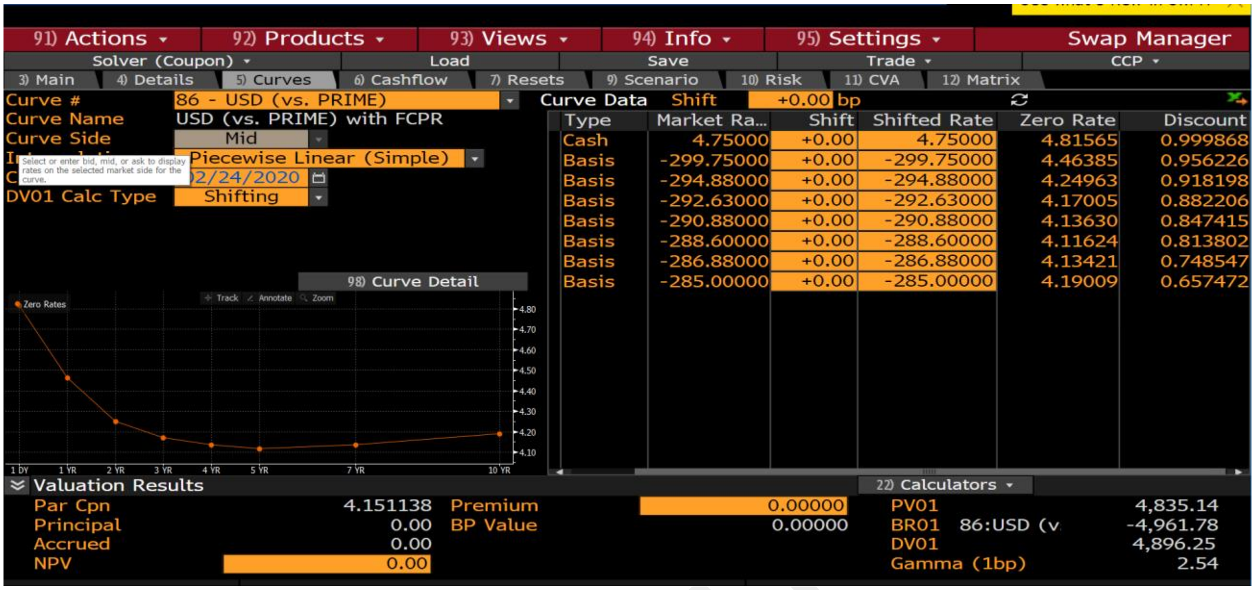
The curves that are used to construct the prime swap curves can be accessed using ICVS 86 function.



		Basis Spread			Basis Adjusted Swap Rates			
Term	Ticker	Pay (bp)	Rcv (bp)	Mid (bp)	Bid	Ask	Mid	Rate Type
1 DY	FCPR	-318.4978	-318.4978	-318.4978	3.2500	3.2500	3.2500	Cash
1 YR	USBF1	-313.8800	-298.8800	-306.3800	3.4416	3.2903	3.3660	Basis
2 YR	USBF2	-308.3800	-293.3800	-300.8800	3.7261	3.5786	3.6524	Basis
3 YR	USBF3	-304.5000	-289.5000	-297.0000	3.9575	3.8105	3.8840	Basis
4 YR	USBF4	-302.8800	-287.8800	-295.3800	4.1025	3.9546	4.0285	Basis
5 YR	USBF5	-301.7500	-286.7500	-294.2500	4.2017	4.0532	4.1274	Basis
7 YR	USBF7	-300.3800	-285.3800	-292.8800	4.3509	4.2028	4.2769	Basis
10 YR	USBF10	-299.0000	-284.0000	-291.5000	4.4726	4.3263	4.3995	Basis

The USBF curve is a swap curve which swaps prime rate into the Libor rate (as shown on the screen below).

The swap curves allow to reduce the prime swap curves to Libor swap curves. The ‘Curves’ tab in the swap tool show the zero rates constructed based on the swap curve.

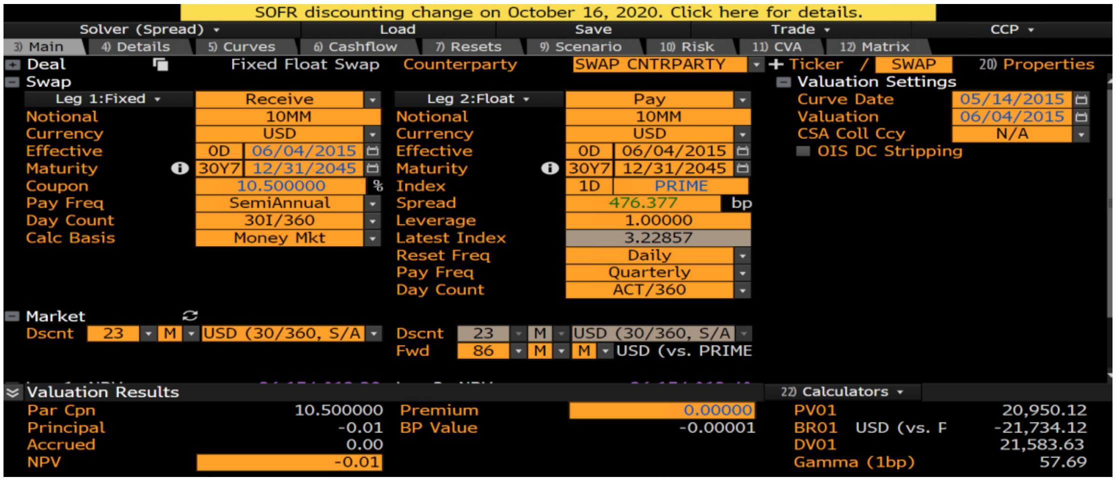


Zero rates are applied as forward prime rates (reset rates) in the floating leg of the cash flow model (as shown in the exhibit below in the print screen of the ‘Resets’ tab).

Reset Date	Reset Rate
04/01/2020	4.64384
04/02/2020	4.64309
04/03/2020	4.64302
04/06/2020	4.64294
04/07/2020	4.64287
04/08/2020	4.64331
04/09/2020	4.64257
04/10/2020	4.64249
04/13/2020	4.64242
04/14/2020	4.64234
04/15/2020	4.64279

A.6.2 Testing Prime rate swap against other base rates

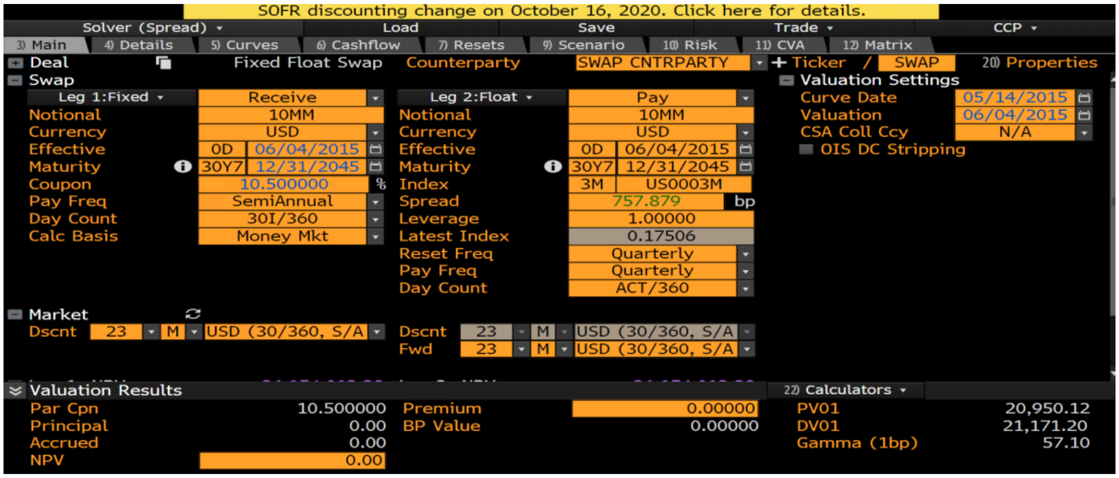
Bloomberg’s SWPM tool print screen for US Prime rate is shown in the exhibit below.



The screen is overall consistent with the swap into the Federal Funds effective rate (which is close to the median of the federal Funds upper and lower bounds[16]). However, the difference in spreads is close to 3.0% (=7.77% - 4.76%) and in practice should be close to 3.125% (see discussion in Appendix G.1.1).

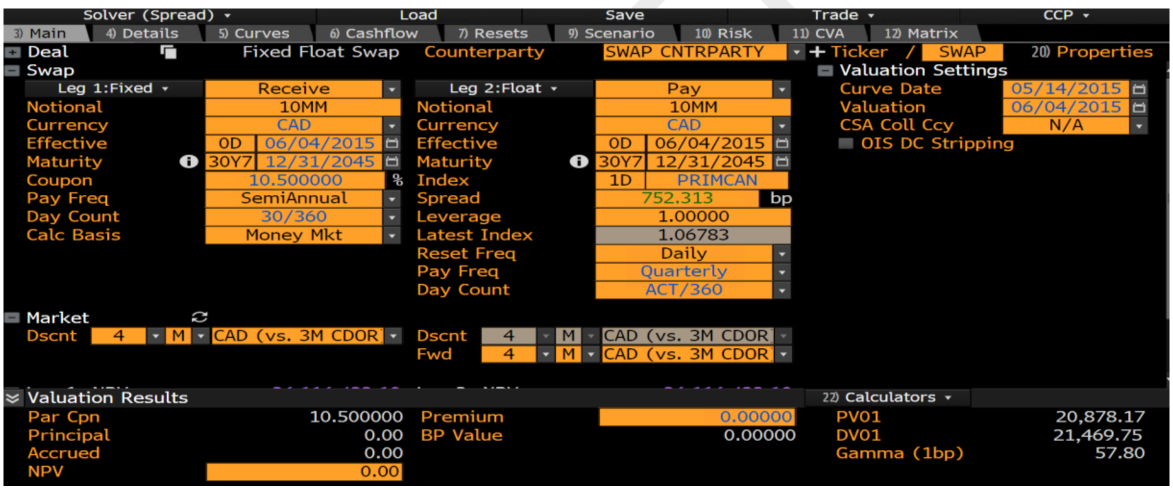
However, the premium between the fixed rate and the Libor rate is higher and includes an additional premium (compared to the premium between fixed and Prime rates). The higher premium is probably due to the fact that Libor rate is ore volatile compared to the FFR or Prime rate.

[16] The spread between the FFR upper and lower bounds is typically 25bps.



The swap to US Libor shows approximately additional 15bps premium for the fixed-rate equivalent (compared to Prime rate swap).

The Bloomberg's SWPM tool was also tested for the Canadian Prime rate but the tool effectively estimates the swap relative to CDOR rate and therefore is not correct.



(Note: the swap to CDOR rate produces a similar 7.52% premium. The tool also shows incorrect 'Latest Index' field value for the Canadian Prime rate).

A.7 Swap to C$ floating rate with US$ Libor base rate

In some cases, a tested floating rate transaction is denominated in one currency (C$) but uses base rate denominated in different currency (US$ Libor). The cross-currency fixed-to-float swap is performed then as follows:

1. A cross-currency fixed-to-fixed (US$-to-C$) swap is performed to estimate the C$ fixed rate.
2. The US$ Libor rate is swapped into the equivalent US$ fixed rate using US$ float-to-fixed swap tool. The US$ fixed rate equivalent of the US$ Libor is referred to as the **reset rate** (following Bloomberg SWPM tool terminology).
3. The US$ Libor spread for the C$ denominated loan is calculated as the difference between the C$ fixed rate and the reset rate.

The steps of the swap estimation approach can be supported by the following argument. By applying the ‘arbitrage’ argument presented in Section 2.1, it can be shown that the reset rate does not depend on the currency in which the loan is denominated. Therefore, if US$ Libor rate is applied to the C$ loan balances (for interest calculations), then the US$ Libor rate can be equivalently replaced by the reset rate and applied to the C$ loan balances. Additional spread between the C$ fixed rate and the reset rate must be added to the reset rate in order to generate the equivalent C$ fixed rate.

Appendix B Bloomberg Swap Curve Data

The section provides a summary of Bloomberg swap curves used in interest rate swap calculations.

B.1 Blomberg swap curves

A few selected swap curves constructed by Blomberg are presented below.

B.1.1 Floating-to-fixed swap curves

The fixed and floating legs of the floating-to-fixed swap curves for selected currencies are described below.

1. **USD** floating-to-fixed Libor swap. The swap curve is described by USSW Bloomberg swap curve with the following swap leg parameters:
   - Floating leg: Reset=Q, Pay Freq=Q, Day Count =Act/360, US Libor-3M (US0003M)
   - Fixed leg: Pay Freq =SA, Day Count =30/360
2. **USD** floating-to-fixed SOFR swap. The swap curve is described by USOSFR Bloomberg swap curve with the following swap leg parameters:
   - Floating leg: Reset=Daily, Pay Freq=A, Day Count=Act/360, US SOFR rate (SOFRRATE)
   - Fixed leg: Pay Freq=Annual, Day Count=Act/360
3. **EUR** fixed-to-floating Euribor swap. The swap curve is described by EUSWxV3 Bloomberg swap curve (where x is the curve maturity term) with the following swap leg parameters:
   - Floating leg: Reset=Q, Pay Freq=Q, Day Count=Act/360, Euribor 3 month (EUR003M)
   - Fixed leg: Reset=A, Day Count=30/360
4. **EUR** fixed-to-floating Euribor swap. The swap curve is described by EUSA Bloomberg swap curve (where x is the curve maturity term) with the following swap leg parameters:
   - Floating leg: Reset=SA, Pay Freq=SA, Day Count=Act/360, Euribor 6 month (EUR006M)
   - Fixed leg: Pay Freq=A, Day Count=30/360
5. **CNY** floating-to-fixed swap. The swap curve is described by CCSH Bloomberg swap curve with the following swap leg parameters
   - Floating leg: Reset=Q, Pay Freq=Q, Day Count=Act/360, Shibor-3M (SHIF3M)
   - Fixed leg: Pay Freq=Q, Day Count=Act/365
6. **CAD** floating-to-fixed swap. The swap curve is described by CDSW Bloomberg swap curve with the following swap leg parameters
   - Floating leg: Reset=Q, Pay Freq=SA, Day Count=Act/365, CDOR-3M (CDOR03)
   - Fixed leg: Pay Freq=SA, Day Count=Act/365
7. **GBP** floating-to-fixed Libor swap. The swap curve is described by BPSW Bloomberg swap curve with the following swap leg parameters
   - Floating leg: Reset=SA, Pay Freq=SA, Day Count=Act/365, GBP Libor-6M (BP0006M)
   - Fixed leg: Pay Freq=SA, Day Count=Act/365

B.1.2 Basis swap curves

The basis swap curve swaps base rates in different currencies. The domestic and foreign currency legs of the basis swap curves for selected currency pairs are described below. The basis swap curves typically use USD currency on leg one.

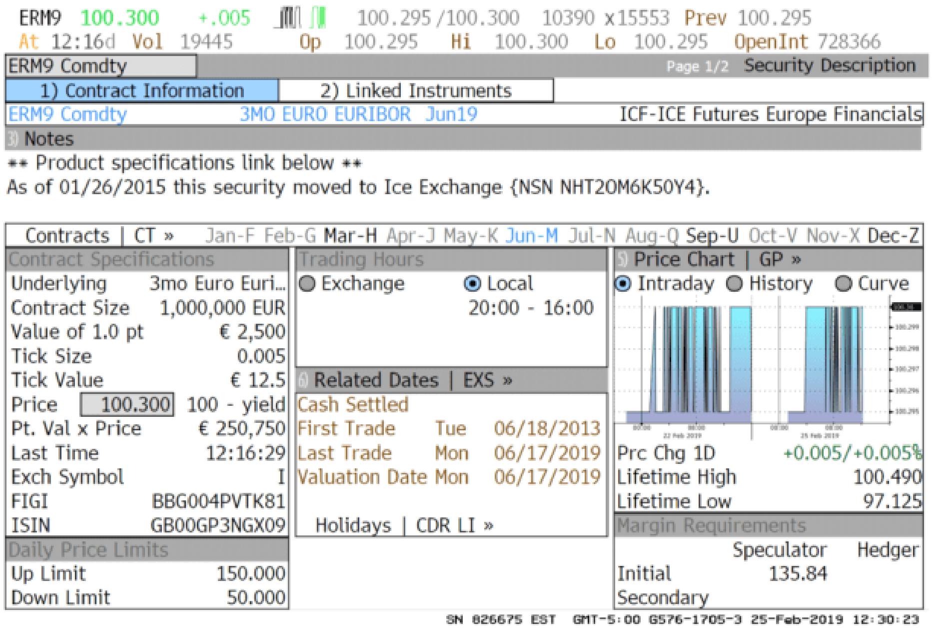
1. **USD-to-CNY** basis swap. The USD-to-CNY basis swap is described by CCBS Bloomberg swap curve with the following swap leg parameters.
   - USD Floating leg: Q, Act/360, US Libor-3M (US0003M)
   - CNY Floating leg: Q, Act/360, Shibor-3M (SHIF3M)
2. **USD-to-CAD** basis swap. The USD-to-CAD basis swap is described by CDBS Bloomberg swap curve with the following swap leg parameters
   - USD Floating leg: Q, Act/360, US Libor-3M (US0003M)
   - CAD Floating leg: Q, Act/365, C$ CDOR-3M (CDOR03)
3. **USD-to-EUR** basis swap. The USD-to-EUR basis swap is described by EUBS Bloomberg swap curve with the following swap leg parameters
   - USD Floating leg: Q, Act/360, US Libor-3M (US0003M)
   - EUR Floating leg: Q, Act/360, Euribor 6 month (EUR003M)
4. **USD-to-GBP** basis swap. The USD-to-GBP basis swap is described by BPBS Bloomberg swap curve with the following swap leg parameters
   - USD Floating leg: Q, Act/360, US Libor-3M (US0003M)
   - EUR Floating leg: Q, Act/365, GBP Libor-6M (BP0003M)

B.2 Bloomberg interest rate forwards

As discussed in section A3, in addition to swap curves Bloomberg uses interest rate forwards to estimate implied FX rates. In this section we illustrate a few examples of how the interest rate forward contracts are presented on Bloomberg.

An example of 6-month forward contract is illustrated below.

Exhibit B.1 Bloomberg screen for 6-month interest rate forward contract



The parameters of the contract are interpreted as follows.

- Contract notional amount is EUR1,000,000 (contract size). At price 100.300, the EUR1mln corresponds to EUR1mln / 100.300 = 9,970 number of contracts;
- The price of each contract is updated whenever the change in price exceeds (in absolute value) the tick value (equal to 0.005). The price of the contract per each tick movement in price is adjusted by 12.5 x price. If the price changes by EUR1, then the contract value changes by (1/0.005) x 12.5 x price = 2,500 x price.
- The 6-month forward rate is estimated as 100 – price = 100 – 100.300 = -0.3%.

B.3 Bloomberg interest rate curves

A list of Bloomberg indices for the interest rate curves typically used in the analysis is described below.

1. Banker's acceptance rates
   - US0003M: US$ 3-month Libor rate
   - US0012M: US$ 12-month Libor rate
   - CDOR3M: C$ 3month CDOR rate
2. Replacement of Libor rates
   - USD: SOFRRATE

     Term SOFR is reported by Bloomberg via TSFR series.

► CAD: CORRA rate

3. Prime rates

   ► PRIME: US national prime rate

   ► PRIMCAN: Canadian national prime rate

4. Central bank policy rates

   ► FEDL01: Federal funds effective rate

   ► : Federal funds target rate

   ► CABROVER: BoC policy rate

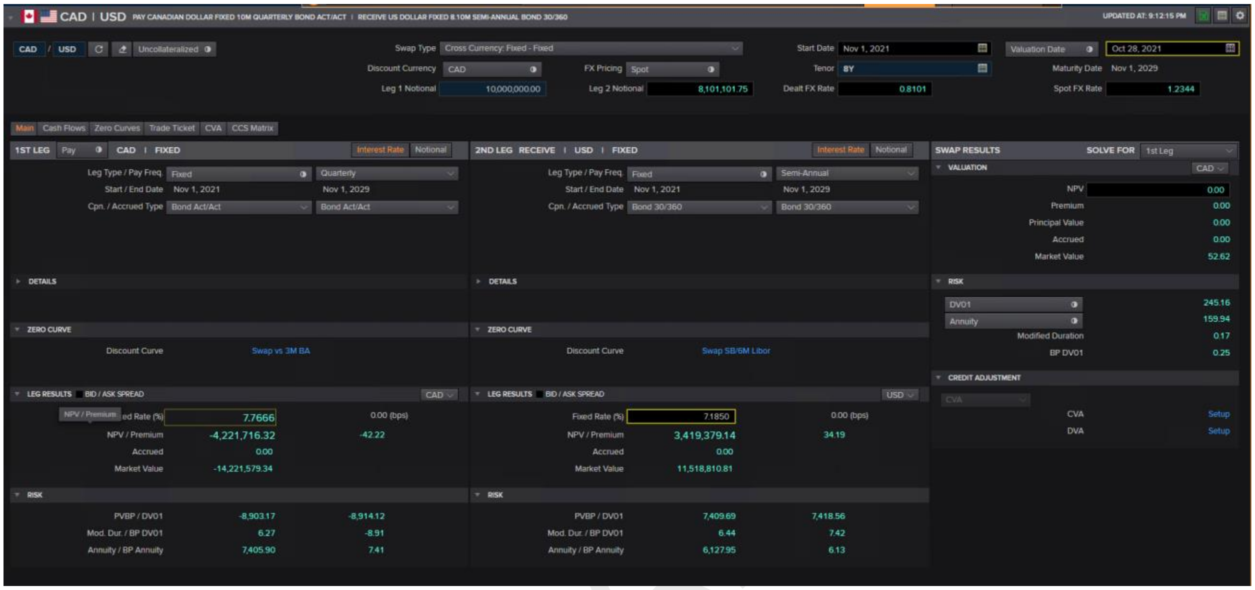
5. Discount curves

   ► USSW: discount curve based on US$ Libor rates

   ► : discount rates based on overnight index rates

Appendix C Eikon Interest Rate Swap Tool

An alternative to Bloomberg SWPM tool, the interest rate swaps can be performed on Eikon as shown in the print screens below. The interest rate swaps are performed using SWAP PRICER tool, which print screen is shown below.



The tools requires selecting the same parameters as the SWPM tool. The FX Pricing need to be set at ‘Spot’ to perform calculations (otherwise N/A will be displayed instead of the output).

Appendix D Eikon Swap Curve API

The swap curves can be obtained from Eikon via Excel API summarized below. The obtained swap curves are also validated against Bloomberg swap curves (discussed in Appendix B.1).

1. USD Libor rate.
2. USD SOFR rate.
3. EUR Euribor rate.
4. CNY Shibor rate.
5. CAD CDOR rate.
6. GBP Libor rate.

Appendix E ac.finance.swap Tool

The ac.finance.swap tool is a swap valuation tool developed as part of this guide. The objective of the tool is converting input curve into an equivalent swapped curve.

E.1 Cross-currency fixed-to-fixed swap

The theoretical equations implemented in the tool are discussed in Section 2. Below is the summary of some general facts about the tool.

- Swap valuation id broken down into two steps: (i) creating swap calculator for a given pair of currencies and all tenors and (ii) running swap for a specific tenor and currency (from the pair of currencies);
- Calculator is neutral to the order of input currencies. For example, if the two currencies are USD and EUR, the calculator can be set either as USD or EUR on leg one (and respectively EUR or USD on leg two). The order will not have any impact on the results of swap valuation.
- The actual currency order will be set to match the currency of the discount factors. For example, if discount factors are denominated in USD, the FX forward curve will be adjusted to match the discount factors (in the previous example, the FX curve will be adjusted as a USD-to-EUR curve. The inputs however can be both either USD-to-EUR or EUR-to-USD FX forward curve).
- The input curves, such as FX and swap curves are assumed to be set at annual frequency. The internal calculations in the tool (such as equations (swp.3), (swp.4), (fx.1), and (fx.2)) are calculated assuming semi-annual frequency. (Parameters may be adjusted.
- Cash flow model is constructed by default using semi-annual frequency (option is also configurable).

Next sections describe inputs used in the tool and the output tables and exhibits produced by the tool.

E.1.1 Tool inputs

The swap tool uses the following inputs and output:

- **Inputs**

  Swap legs parameters. Parameters of the two legs of the swap, which specify currency, interest payment frequency, and day count basis on each leg;

  Maturity. Swap maturity parameter;

  FX forward rates. Direct or implied FX forward curve;

  Discounts. Discount factor curve;

  Input curve. Input curve which needs to be swapped into a new curve with the same maturity parameter and different interest rate parameters (currency, frequency, day count).

- **Output**

  Implied FX forward rates. Forward rates used in the swap calculations. The rates are either direct or implied depending on the input curves parameters.

  Output curve. Output swapped curve.

Cash flows. Swap cash flows constructed for a specific date.

The tool creates a swap calculator, which is applied then to swap yield curves from one currency into another. Note that for a swap calculator it does not matter which currency is set on leg one and which is set on leg two. Therefore, it is recommended always set USD on the first leg of the swap.

E.1.2 Tool output

E.2 Swap curve estimation

The algorithms implemented by the tool are discussed in Appendix H.

Appendix F Validation of the ac.finance.swap Tool Output

The section summarizes the validation analysis performed for the swap valuation using ac.Swap tool.

F.1 Internal validation

The internal validation tests the following elements of the swap valuation output.

- **Currencies**. Validate that the leg one and leg two swap currencies match the FX forward curve currencies. Note that the swap calculator currencies are set for reference purpose only. The calculator identifies the currencies and the direction of the swap from the FX forward curve currencies. However, to ensure that the user set the currencies in the FX forward curve correctly, the user is enforced to set also the swap calculator currencies, which are used to validate the FX forward curve currencies;
- **Coefficients**. Review the estimated $a_{0,T}$ and $a_{1,T}$ coefficients of the swap calculator. In practice, the coefficient $a_{1,T}$ is expected to be close to one in most cases. The validation tool returns the minimum, maximum, and average values of the $a_{1,T}$ coefficient.
- **Approximations**. Estimate the difference between the approximate and exact swap calculations. The approximate calculations refer directly to the FX forward rates (and interest rate parity) and therefore have a simple interpretation. If the deviation of the exact from the approximate calculations is material, the source of deviation shall be identified and reviewed.
- **Cash flow model**. A direct validation of the model is performed by estimating the leg one and leg two cash flows of the swap. The NPV of the difference between the leg one and leg two cash flows must be equal to zero.

Example of internal validation analysis is illustrated below.

F.2 Validation using Bloomberg SWPM tool

Validation against Bloomberg SWPM tool includes review of the following items.

- **Discounts**. Review of discount rates;
- **FX forwards**. Review of the FX forward rates;
- **Cash flows**. Review of the swap cash flows.

Example of the above elements of the validation analysis is illustrated below.

Appendix G Historical Behavior of Interest Rates

Market forward rates are available only for specific base rates such as 3-month Libor. In practice, the interest rates often need to be swapped into Prime or other base rates. The extension of the forward rates to other base rates is performed based on the correlation in the interest rate movements.

The following behavior of interest rates is tested in this section.

1. **US Prime** rate equals to the federal funds rates plus **3%** premium.
2. **Canadian Prime** rate equals to the Bank of Canada (**BoC**) policy rate plus **2.2%** premium.
3. The US federal funds and BoC policy rates are either fixed for extended period of times or are updated with step 0.25% when the government adjusts market interest rates through open market operations. The deviation of the Prime from the policy rate is observed primarily due to the fact that the Prime rate is typically adjusted a few days in advance prior to the policy rate movement. After correcting for a few days difference between the rate adjustment, the 3% and 2.2% premiums are observed consistently throughout the 2000 – 2020 history of interest rates.
4. Many revolving loan agreements set the terms such that Prime and Banker's Acceptance advances can be drawn from the loan borrowing limit. The spread between the margins of the Prime and BA loan advances is set at **1%**.
5. The loans which allow to draw advances in both US$ and C$ currencies typically set the same margin on both the US$ and C$ loan advances.
6. The rational for the same margin on both the US$ and C$ loan advances can be tested empirically as follows: (i) swap US$ fixed rate into the US$ floating rate (using USSW swap curve); (ii) swap US$ fixed rate into C$ fixed rate and then swap C$ fixed into C$ floating rate (using CDSW curve); (iii) compare the estimated margins on the US$ and equivalent C$ rates.
7. The following arbitrage equation is closely approximated with historical data: FX swap from US-to-Canada fixed rates applied to USSW rates equals to the following composition of swaps: (i) US fixed-to-float swap (described by USSW series), (ii) Libor to CDOR base swap (described by , and (iii) CAD float-to-fixed swap 9described by CDSW series).

G.1 Prime rate premium

The section discusses the historical behavior of premium between the Prime and the central bank policy rates, which is applied to estimate fixed-to-floating swaps with Prime used as a base rate.

G.1.1 Historical Prime premium patterns

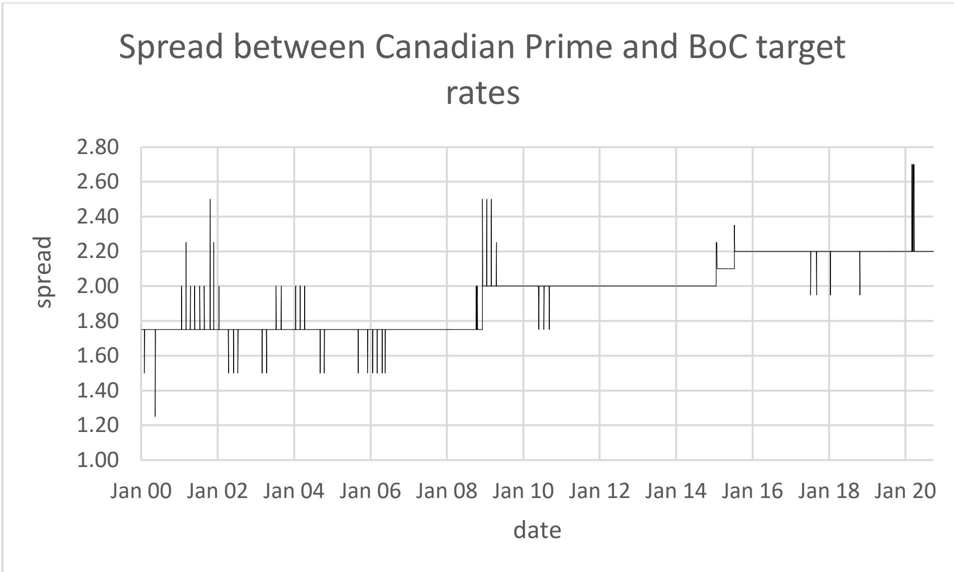
The spread between the Prime and policy rates (denoted as s_t) was estimated using the following equation:

$$s_{t+1} = s_t + \left(i_{t+1}^{Prime} - i_t^{Prime}\right) - \left(i_{t+1}^{policy} - i_t^{policy}\right) \quad \text{(G.1)}$$

where i_t^{Prime} and i_t^{policy} are respectively the Prime and the policy rates. Whenever there is deviation in the prime rate from the current rate, $i_{t+1}^{Prime} \neq i_t^{Prime}$, the spread deviates from the target spread value. The movement in the Prime rate is typically caused by the change in the policy rate. The change in the policy rate is typically observed a few days later after the change in the Prime rate. After the policy rate is adjusted, the spread returns to its target value.

As can be observed from the exhibits below, in addition to temporal deviations from the constant spread, there is also a structural change in the target spread. In the case of the spread between Canadian Prime and BoC policy rate, the spread changed over time from 1.75% to 2.0% and finally to 2.2%.

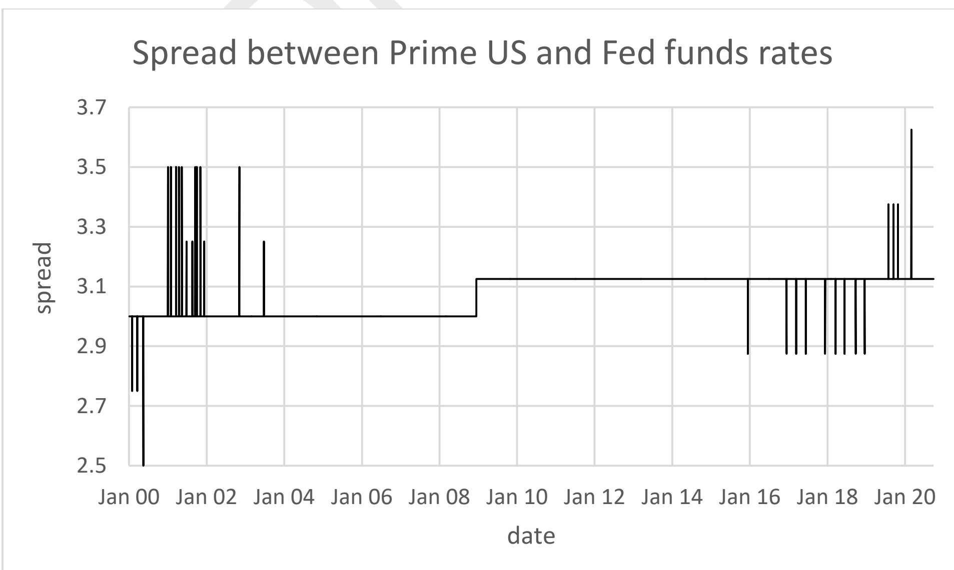
Exhibit G.1 Change in the spread between Canadian Prime and BoC policy rates



The exhibit illustrates that a structural change in the spread from 1.75% to 2% occurred in October 2008 and from 2.0% to 2.2% in January – June 2015.

The spread between the US Prime and federal funds target rate is presented in the exhibit below.

Exhibit G.2 Change in the spread between US Prime and federal funds target rates



The exhibit illustrates that a structural change in the spread from 3.0% to 3.125% occurred in December 2008.

Conclusion: For swap calculation purposes, Prime rate may be viewed as having a fixed premium over the central bank policy rate. The policy rate can be approximated using Bankers Acceptance or another short-term risk-free rate.[17]

G.1.2 Prime rate premium observed in loan agreements

Many loan agreements use Prime and Bankers' Acceptance rates to (i) either allow drawing advances with the interest rate set either based on the Prime or BA rates, or (ii) define the base rate using a combination of Prime and BA rates. The two alternative terms of the loan agreement are illustrated below.

1. In the first alternative, the loan agreement specifies separately Prime rate advances and BA rate advances. Depending on what type of advance is drawn by the borrower, a different margin applies. An example of the 'applicable margin' term specified for the Prime and BA advances is illustrated below.

   > "**Applicable Margin**" means, with respect to Advances under the Revolving Facility, (i) 0.45% for Prime Rate Advances, and (ii) 1.45% for Advances by way of Bankers' Acceptances.

   Since the Prime rate and BA rate advances can be drawn at the option of the borrower, the 1% spread between the margins can be applied to convert the swap spread over the BA rate into the equivalent swap spread over the Prime rate.

2. Alternatively, the loans can define the base rate as the maximum of (i) the Prime rate, (ii) the BA rate plus premium, or (iii) the policy rate plus premium. The margin in the loan agreement is specified with respect to the base rate. Formally, this type of loan agreement does not provide an option to borrow either using Prime or BA rate advances (and, therefore, the margin over the Prime rate cannot be interpreted as equivalent to the margin over the BA rate). However, the premium added to the BA rate is indicative of the spread between the Prime and BA rates and is consistent with the margin spread specified in the loan agreements discussed in the previous item. An example of 'base rate' definition is illustrated below.

   > "Base Rate" means for any day a fluctuating rate per annum equal to the highest of (a) the rate of interest in effect for such day as publicly announced from time to time by Bank of America as its "prime rate"; (b) the Federal Funds Rate for such day, plus 0.50%; and (c) the LIBOR Rate for a one month interest period as determined on such day, plus 1.00%. The

Conclusion: the examples above illustrate that the loan agreements typically assume **1% margin premium** for the BA rate advances compared to the Prime rate advances. Note that the 1% margin premium typically does not compensate for the 2-3% spread between the BA and Prime rates observed consistently in historical data and, therefore, it is always more costly to borrow Prime rate advances. However, the 1% margin premium is observed consistently in all loan agreements.

G.1.3 Fixed-to-float swap with the Prime base rate

Based on the above overview of the Prime rate historical behavior and terms of the loan agreements with both Prime rate and BA rate advances, the following methodology can be applied to perform foxed-to-float swap with the Prime rate used as the base rate.

1. Fixed-to-float swap based on Prime historical pattern. The swap valuation is described as follows:
   - swap fixed rate into the equivalent spread over BA rate;
   - subtract 2% premium from the estimated spread to produce an equivalent spread over Prime rate.

[17] Note that Bloomberg's SWPM tool allows to select the US Federal Funds rate (FFR) as the base rate.

2. Fixed-to-float swap based on the spread between Prime/BA rate advances in loan agreement. The swap valuation is described as follows:
   - swap fixed rate into the equivalent spread over BA rate;
   - subtract 1% premium from the estimated spread to produce an equivalent spread over Prime rate

G.2 Consistency between spreads on CDOR and Libor loans

This section tests the following equivalence between the spreads over US$ Libor and C$ CDOR rates. Similar to the previous section, the equivalence is assessed from two perspectives: (i) based on estimate of historical patterns, and (ii) based on review of legal agreements.

G.2.1 Historical discrepancies between US$ Libor and CDOR spreads

The discrepancy between the US$ Libor and CDOR spreads was estimated as follows.

1. The fixed i^{US} rate is converted to spread over US$ Libor s^{Libor} as follows: $s^{Libor} = i^{US} - i^{ussw}$, where i^{ussw} is the swap rate (reported by Bloomberg USSW curve);
2. The fixed i^{US} rate is converted to fixed C$ rate i^{CA} using cross-currency fixed-to-fixed swap tool.
3. The fixed i^{CA} rate is converted to spread over C$ CDOR s^{CDOR} as follows: $s^{CDOR} = i^{CA} - i^{CDsw}$, where i^{CDsw} is the swap rate (reported by Bloomberg CDSW curve).
4. The spreads s^{Libor} and s^{CDOR} are compared for consistency.

The results of the analysis are described below.

<u>*Conclusion*</u>:

G.2.2 Consistency between US$ Libor and CDOR spreads based on review of loan agreements

Loans issued by companies which have operations in both US and Canada typically allow to borrow in both US$ and C$ currencies. The interest rate margins in such loans are specified for both US$ Libor and CDOR base rates. In practice, the loans set the same margin for both US$ and C$ advanced. An illustration of US$ and C$ margin terms in a multi-currency loan agreement is illustrated below.

<u>*Conclusion*</u>: based on the review of loan agreements, it can be concluded that the spread estimated for the US$ denominated floating rate loan can be equivalently applied as a spread for the C$ denominated floating rate loan.

G.3 Consistency between implied and direct forward rates

The consistency between the implied and direct forward rates was tested as follows:

1. The implied FX rates were estimated based on the US$ and C$ swap curves (USSW and CDSW) and US$-to-C$ basis swap curve.
2. The direct forward rates were estimated based on the C$ FX currency forward rates.

3. The implied and direct FX rates were compared to each other.

Alternatively, the alternative approaches were compared by reviewing how the cross-currency fixed-to-fixed swap depends on two alternative swap estimation approaches. The steps of the swap consistency testing are summarized as follows.

1. The US$ fixed rates are swapped to C$ fixed rates using C$ FX forward curves;
2. The US$ fixed rates are converted to US$ spreads over Libor (using USSW curve); US$ Libor is converted to C$ CDOR using US$-to-C$ basis swap curve; the C$ CDOR is converted to C$ fixed rates (using CDSW swap curve).
3. The C$ fixed rates estimated under two alternative approaches are compared to each other.

The results of the analysis are summarized below.

Conclusion:

G.4 Consistency between Libor and federal funds term structures

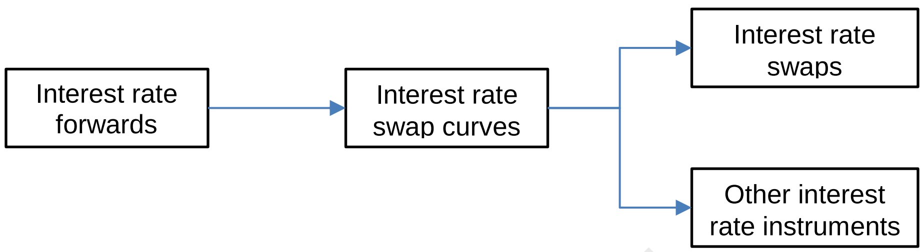
The purpose of the exercise is to verify that the central bank policy rates and BA rates follow effectively the same interest rate process with a fixed constant premium charged in the BA rates. The hypothesis was tested for the US$ federal funds and Libor rates since Bloomberg allows to preform swap for both base rates and, therefore, estimates the forward rates in both cases. As discussed in Appendix H.3, the forward rates can be applied to estimate respective swap curves and related term structures of interest rates.

In the exercise described below, the federal funds and Libor forward rates were estimated for 20 different dates selected on 31 January of years 2000 through 2020. The forward rates were converted then to swap rates and compared to each other.

Conclusion:

Appendix H Swap Curve Estimation

Swap curves are used as input in the swap valuation tool. In practice swap curves are estimated based on interest rate forward traded in the market. The interest rate forwards are the base instruments that are used to price other interest rate instruments. A swap curve can be viewed as a convenient pricing instrument. Schematically it can be described by the following diagram.



This section provides a high-level overview of interest rate forwards and a description of how the swap curves are estimated based on the interest rate forwards. Forward rates can be viewed as basis to perform valuation of more complex swap instruments. However, forward rates are typically not available directly. A common practice is to report swap curve, which can be used to recover forward rates and estimate other (derivative) swap curves (as discussed in Section **Error! Reference source not found.**).

H.1 Interest rate futures

Definition: A short-term interest rate (**STIR**) future is a futures contract that derives its value from the interest rate at maturity. Common short-term interest rate futures are Eurodollar, Euribor, Euroyen, Short Sterling and Euroswiss, which are calculated on LIBOR at settlement, with the exception of Euribor which is based on Euribor. This value is calculated as 100 minus the interest rate. Contracts vary, but are often defined on an interest rate index such as 3-month sterling or US dollar LIBOR.

Interest rate futures are used to hedge against the risk that interest rates will move in an adverse direction, causing a cost to the company. For example, borrowers face the risk of interest rates rising. Futures use the inverse relationship between interest rates and bond prices to hedge against the risk of rising interest rates. A borrower will enter to sell a future today. Then if interest rates rise in the future, the value of the future will fall (as it is linked to the underlying asset, bond prices), and hence a profit can be made when closing out of the future (i.e. buying the future). Treasury futures are contracts sold on the Globex market for March, June, September and December contracts. As pressure to raise interest rates rises, futures contracts will reflect that speculation as a decline in price. Price and yield will always be in an inversely correlated relationship.[18]

Definition. A forward rate agreement (**FRA**) is a contract which specifies the difference between a fixed rate and a floating rate on a specific index such as Libor, Euribor, etc. Informally the value of the FRA is $C = N \times d \times \frac{R-r}{1+dr}$, where N is notional amount, d is day count calculated for effective and maturity dates of the contract, R is the fixed rate, r is the floating index rate, and $\frac{1}{1+dr}$ is the discount factor.

[18] https://en.wikipedia.org/wiki/Interest_rate_future

“A forward rate agreement (FRA) is a tailor-made, over-the-counter financial futures contract on short-term deposits. A FRA transaction is a contract between two parties to exchange payments on a deposit, called the Notional amount, to be determined on the basis of a short-term interest rate, referred to as the Reference rate, over a predetermined time period at a future date. FRA transactions are entered as a hedge against interest rate changes. The buyer of the contract locks in the interest rate in an effort to protect against an interest rate increase, while the seller protects against a possible interest rate decline. At maturity, no funds exchange hands; rather, the difference between the contracted interest rate and the market rate is exchanged. The buyer of the contract is paid if the published reference rate is above the fixed, contracted rate, and the buyer pays to the seller if the published reference rate is below the fixed, contracted rate. A company that seeks to hedge against a possible increase in interest rates would purchase FRAs, whereas a company that seeks an interest hedge against a possible decline of the rates would sell FRAs”.[19]

H.2 Overview of swap curve estimation methods

A general approach to estimate a custom swap curve based on the swap curve inputs (estimated for example by Bloomberg) is illustrated in the diagram below.

Exhibit H.1 Custom swap curve estimation diagram

This section discusses the steps applied to construct different types of swap curves. Specifically,

1. Construction of the float-to-fixed swap curve based on forward rates;
2. Estimation of forward rates based on the float-to-fixed swap curve;
3. Estimation of float-to-float swap curve, where the two legs are denominated in the same currency but have different frequencies

H.3 Estimation of float-to-fixed swap curves

Suppose that i_t^{fwd} are base rate forward rates and i_t^{swp} are the swap curve rates which were estimated based on the i_t^{fwd} forward rates. The NPV of the cash flows in the fixed and floating legs of the swap are equal and are described by the following equation.

(H.1) $$i_T^{swp} \times \sum_{t=1}^{T} D_t = \sum_{t=1}^{T} i_t^{fwd} \times D_t$$

The swap rates are described as the weighted-average of the forward rates using the following equation.

[19] https://en.wikipedia.org/wiki/Forward_rate_agreement

(H.2)

$$i_T^{swp} = \sum_{t=1}^{T} i_t^{fwd} \times w_t$$

where

(H.3)

$$w_t = \frac{D_t}{\sum_1^T D_s}$$

are the applied weights estimated based on the discount rates.

H.4 Estimation of forward rates

The forward rates are estimated recursively from the equation (H.1) using the following equation

(H.4)

$$i_T^{fwd} = i_T^{swp} + \frac{1}{D_T} \sum_{t=1}^{T-1} \left(i_T^{swp} - i_t^{fwd} \right) \times D_t$$

where the first forward rate is set equal to the base rate $i_1^{fwd} = i^{base}$. If the frequency of the swap curve and the forward rates don't match, the swap curve is interpolated to match the frequencies.

H.5 Bloomberg reset rates

H.6 Examples of custom swap curve estimation

Appendix I Notes to Equations

The section provides additional technical details for interest rate swap calculations.

I.1 Implied FX forward rates

In this section we present the equations for the implied FX forward rates based on (i) general term structures and (ii) zero-coupon term structures of domestic and foreign currency denominated interest rates .

I.1.1 General term structure

Suppose that c_T^A and c_T^B are interest rates on debt transactions that have comparable terms with exception of the debt currency. The two interest rate curves are estimated either (i) based on basis swap curves or (ii) samples of sovereign debt transactions.

The interest rate relationship was derived in equation (2.5) of Section 2.2.1 is described by the following equation:

$$c_T^B = \frac{1}{\left[\sum_{t=1,\dots,T} D_t \times \frac{S^{A,B}}{F_t^{A,B}} \times \Delta_t\right]} \times \left(D_T \times \left(1 - \frac{S^{A,B}}{F_T^{A,B}}\right) + c_T^A \times \sum_{t=1,\dots,T} D_t \times \Delta_t\right)$$

or equivalently

$$\left[\sum_{t=1,\dots,T} D_t \times \frac{S^{A,B}}{F_t^{A,B}} \times \Delta_t\right] \times c_T^B = D_T \times \left(1 - \frac{S^{A,B}}{F_T^{A,B}}\right) + c_T^A \times \sum_{t=1,\dots,T} D_t \times \Delta_t$$

After rearranging the terms, the equation can be represented as follows

$$D_T \times \frac{S^{A,B}}{F_T^{A,B}} \times (1 + \Delta_t \times c_T^B) = D_T \times (1 + \Delta_t \times c_T^A) + \sum_{t=1,\dots,T-1} D_t \times \left(c_T^A - c_T^B \times \frac{S^{A,B}}{F_t^{A,B}}\right) \times \Delta_t \qquad \text{(I.1)}$$

The above equation presents the relationship between period T FX forward rate and the FX forward rates estimated for periods $t = 1, \dots, T-1$. The equation is estimated sequentially using the underlying term structures of interest rates.

If we denote

$$f_t^{A,B} = \frac{F_t^{A,B}}{S^{A,B}}$$

then the equation can be equivalently represented as follows

$$f_T^{A,B} = \frac{D_T \times (1 + \Delta_t \times c_T^B)}{D_T \times (1 + \Delta_t \times c_T^A) + A_{t-1}} \qquad \text{(I.2)}$$

where

(I.3)
$$A_T = \sum_{t=1,\dots,T} D_t \times \left(c_T^A - \frac{c_T^B}{f_t^{A,B}}\right) \times \Delta_t = A_{T-1} + D_T \times \left(c_T^A - \frac{c_T^B}{f_T^{A,B}}\right) \times \Delta_T$$

I.1.2 Zero-coupon term structure

The swap premium estimated for zero-coupon bonds can be used as a proxy for the swap premium estimated for bonds with generic coupon structure. The zero-coupon swap premium/discount equation derived in this section can also be used to derive implied forward rates.

Suppose that $c^A = 0, c^B = 0$ and P^A and P^B are respectively prices of zero-coupon bonds denominated in currencies A and B. Then the equation derived in the section above can be presented as follows

$$D_T = D_T \times \frac{P^A}{P^B} \times \frac{S^{A,B}}{F_T^{A,B}}$$

where $P^A = D_T$ is the price of zero-coupon bond denominated in currency A. The equation relates the value of one unit of cash received in period T and denominated in currency A to the same unit of cash, which is replicated by (i) converting the price P^A of the instrument into currency B; (ii) purchase of zero-coupon bond denominated in currency B; and finally (iii) using forward FX rate to convert the unit of cash denominated in currency B into a unit of cash denominated in currency A.

The above equation can be represented equivalently as

$$P^B = P^A \times \frac{S^{A,B}}{F_T^{A,B}}$$

If the currency forward rate is above the spot rate, $F_T^{A,B} > S^{A,B}$, then the zero-coupon bond denominated in currency B is traded at discount compared to the zero-coupon bond denominated in currency A ($P^B < P^A$).

The equation can be represented equivalently in terms of the yield rates:

$$\frac{1}{(1+y^B)^T} = \frac{1}{(1+y^A)^T} \times \frac{S^{A,B}}{F_T^{A,B}}$$

or

(I.4)
$$y^B = (1+y^A)\left(\frac{F_T^{A,B}}{S^{A,B}}\right)^{\frac{1}{T}} - 1 \approx y^A + \left[\left(\frac{F_T^{A,B}}{S^{A,B}}\right)^{\frac{1}{T}} - 1\right] = y^A + f_t^{A,B}$$

where

(I.5)
$$f_T^{A,B} = \left(\frac{F_T^{A,B}}{S^{A,B}}\right)^{\frac{1}{T}} - 1$$

is T –period forward rate.

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The equation can be used to (i) derive an approximation of the swap premium/discount (note that it requires only spot rates and forward rates at maturity term and does not require the full term structure of the forward rates); and (ii) derive implied forward rates with maturity T from zero-coupon yield rates.

Appendix J Examples

In this section we discuss some specific examples of interest rate swaps.

J.1 Multi-currencies and inflation-adjusted interest rates

For certain countries, the interest rates can be quoted in different currencies, which may produce very different interest rate swap results. Alternative currencies are used for the purpose of reducing the impact of the inflation on interest rates. Alternatively, the interest rate can be quotes as index-linked rate, where index is measuring country inflation rate. A special attention in the interest rate swap calculations should be given to those regions.

J.1.1 Chile

J.1.2 China

J.1.3 Brazil

Appendix K References

List of references is provided below.

[1] John C. Hull, “Options, Futures, and Other Derivatives”, 7th edition, Pearson, Prentice Hall, 2009