VALUATION OF CMO SECURITIES

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List of Abbreviations

The following abbreviations and symbols are used in this guide:

ac.finance.CMO	Excel/java based CMO valuation tool developed as part of this guide
AS	Accelerated Security bond receives principal payments faster than its respective collateral.
Bloomberg	
CDS	Credit default swap
CMO	Collateralized mortgage obligation
CSTR	Collateral strip rate is a bond which interest is based on the weighted net interest of the collateral
EXCH	Exchangeable bond is a bond which is exchangeable with a combination and / or proportion of other bonds. The exchangeable bond is entitled to receive a proportionate amount of cash flow from the related bond combination
Fannie Mae	Federal National Mortgage Association
FICO	Fair Isaac Corporation,
FLT	Floating-rate bonds
Freddie Mac	Federal Home Loan Mortgage Corporation
INV	Inverse floating-rate bond which coupon rate varies inversely with the changes on a base index rate.
IO	Interest-only bond receives some or all of the underlying collateral interest and receives no or little of principal. The IO bond notional amount is used to calculate the interest amount. The IO bond nominal amount is the actual stated principal amount which will be paid to the bond. It is typically extremely small compared to other classes.
NAS	Non-accelerated Security bond receives principal payments more slowly that its respective collateral. NAS bonds are typically linked to AS bonds which receive accelerated principal payments (which otherwise would be allocated to NAS bonds)
NPV	Net present value
NTL	Notional principal bond which is a pure IO bond with no actual principal
PAC	Planned amortization class
PIP	Prepayment Interest Penalty bond is designated to receive mortgage loans' prepayment premiums or charges
PO	Principal-only bond which does not receive any interest payments
PSA Prepayment Model	A prepayment scale developed by the Public Securities Association in 1985 for analyzing American mortgage-backed securities
R	Residual bond is designated for the tax purposes as the residual interest in a REMIC structure
REMIC	A real estate mortgage investment conduit

RMBS	Residential mortgage-backed security
RSTP	Ratio Strip bond receives fixed percentage of each loan principal. The percentage is unique for each loan and is based on the loan's net rate.
SEQ	Sequential pay bonds which start to pay when classes with an earlier priority balances were reduced to zero. SEQ bond principal may share the principal pay down on a pro rata basis with another class
SSNR	Super Senior bond has higher principal and/or interest distribution priority over other senior bonds in a deal
SSUP	Senior Support bond has a subordinated principal and/or interest distribution priority to other senior bonds in a deal
SPE	Special purpose entity
SUB	Subordinated bond which principal and / or interest distributions are subordinated to senior bonds. The purpose of subordinated bonds is to provide credit enhancement to senior bonds.
TAC	Targeted amortization class
WAC	Weighted-average coupon
WALA	Weighted-average loan age
WAM	Weighted-average maturity

Section 1: Introduction

A collateralized mortgage obligation (**CMO**) is a type of complex debt security that repackages and directs the payments of principal and interest from a collateral pool of mortgage loans to different types of securities. Legally, a CMO is a debt security issued by a special purpose entity (**SPE**) (thus it is not considered a debt owed by the institution creating and operating the SPE). The SPE is the legal owner of a set of mortgages, called a pool. Investors in a CMO buy bonds issued by the entity, and they receive payments from the income generated by the mortgages according to a defined set of rules. With regard to terminology, the mortgages themselves are termed collateral, 'classes' refers to groups of mortgages issued to borrowers of roughly similar credit worthiness, tranches are specified fractions or slices, metaphorically speaking, of a pool of mortgages and the income they produce that are combined into an individual security, while the structure is the set of rules that dictates how the income received from the collateral will be distributed. The legal entity, collateral, and structure are collectively referred to as the deal. Unlike traditional mortgage pass-through securities, CMOs feature different payment streams and risks, depending on investor preferences.

Servicer

Lead manager

Trustee

CMO securities offer the following advantages.

1. An average mortgage loan amount (say US$ 300,000) is quite material for an average investor. Issue of bonds collateralized by mortgage loans allows to set flexible principal amount on each bond. For example a US$ 300,000 mortgage loan can be split into 300 bonds with US 1,000 principal amount.
2. Mortgage loans are long-term (they typically have a 30-year maturity term). Investors typically prefer not to lock their funds on such a long period. CMO instruments are traded publicly and therefore allow investors to sell the bonds at any time.
3. Securities issued in a CMO deal typically have a flexible structure so that investors with different risk / return preferences can buy the securities.
4. Default risk s diversified in a pool of mortgage loans. Traditionally the CMO securities were viewed as having very good creditworthiness. However the market crash in 2008-2009 showed that defaults in a mortgage loan pool can be strongly correlated and collateral value of the houses can decrease significantly during the crisis. The credit rating on many CMO securities was downgraded from AA to CCC within a few month period during the market crash. Therefore potentially investment into CMO securities can be highly risky.

The major risks associated with the CMO securities are the following:

- **Loss due to default risk**. House buyers default on their mortgages. After the default, the houses go into foreclosure and may often be sold at the price that is lower than the mortgage loan outstanding principal balances.
- **Prepayment (refinancing/reinvestment) risk**. Mortgage loans are amortized over the 30-year maturity term. Moreover the loans can be prepaid without penalty partially or in whole at any time. The mortgage loans are often prepaid when the mortgage market rates go down so that the mortgage can be refinanced at a lower rate. The risk is called the interest rate risk (or refinancing

risk). Effectively the investment horizon in the CMO securities is less than 30-years. The prepayment risk must be reflected as the yield premium or price discount of the CMO security.

To address the above risks, the CMO deal structure specifies loss and principal allocation rules so that different securities within the CMO deal are protected differently against the default and prepayment risks.

1.1 Terminology

The following terminology is used in the guide.

- **REMIC**. A real estate mortgage investment conduit (REMIC) is "an entity that holds a fixed pool of mortgages and issues multiple classes of interests in itself to investors" under U.S. Federal income tax law and is "treated like a partnership for Federal income tax purposes with its income passed through to its interest holders". REMICs are used for the pooling of mortgage loans and issuance of mortgage-backed securities and have been a key contributor to the success of the mortgage-backed securities market over the past several decades.
- **CMO deal**. A CMO deal term (or simply CMO) refers to a collection of CMO securities issued by a specific SPE with a structure of interest/principal/loss allocations defined in the CMO deal prospectus and pool of mortgages used as a collateral for CMO deal interest and principal allocations.
- **CMO series**. Same as CMO deal.
- **CMO tranche**. CMO tranches describe the structure of a CMO deal.
- **CMO class**. Same as CMO tranche.
- **CMO group of classes**. For some CMO deals, the collection of CMO classes can be divided into multiple (usually two) groups. The pool of loans is often also divided into loan subsets so that the interest payments as well as loss and principal distributions from loan pool subsets are allocated to the related tranche group. The groups are typically labelled as 1A1, 1A2, …; 2A1, 2A2, …, etc.
- **CMO security**. A specific CMO bond issued as part of CMO deal. Each CMO security is issued based on specific CMO tranche. The allocation of interest/principal/losses to the CMO security is determined by the allocation rules for a specific CMO tranche.
- **CMO bond**. Same as CMO security.
- Residential mortgage-backed security (**RMBS**) is used interchangeably with CMO security term.
- **Delinquency rates**. A delinquency rate is the percentage of loans within a loan portfolio that have delinquent[1] payments. A delinquency rate can be further broken down by categories. It is common for lenders to provide delinquency levels by both length of delinquency and delinquency by credit quality category.

  Typically a loan is reported as delinquent after 2 consecutive missed payments (60 days). The loan may be continued to be reported delinquent up to 270 days (with the delinquency status updated every month). After 270 days of late payments the US code of federal regulations considers the loan to be in default state.
- **Tranche credit support**.
- **Senior credit support depletion date** is the date when the balances of all junior and mezzanine securities are reduced to zero.
- **Crossover date**. Same as senior credit support depletion.

[1] In finance, an individual or entity is delinquent upon failure to make contractually obligated debt payments in a regular, timely manner.

- **Pro-rata allocation structure** is the structure which assumes allocation to each tranche proportional to the tranche outstanding balances.
- **Attachment point** is the amount of losses in the covered CMO security at which the coverage by the CDS contract starts. Attachment point is typically measured as a percentage of the security outstanding balance and changes over time as the security outstanding balances change.
- **Detachment point** is the amount of losses in the covered CMO security at which the coverage by the CDS contract stops. Detachment point is typically measured as a percentage of the security outstanding balance and changes over time as the security outstanding balances change.
- **Credit enhancement**. Credit enhancement is the improvement of the credit profile of a structured financial transaction or the methods used to improve the credit profiles of such products or transactions.

  Establishing a senior/subordinated structure is one of the most popular techniques to create internal credit enhancement. Cash flows generated by assets are allocated with different priorities to classes of varying seniorities. The senior/subordinated structure thus consists of several tranches, from the most senior to the most subordinated (or junior). The subordinated tranches function as protective layers of the more senior tranches.

  The tranche with the highest seniority has the first right on cash flow. Such protection comes under a waterfall structure. Priority for cash flow comes from the top, while distribution of losses rises from the bottom. If an asset in the pool defaults, the losses thus incurred are allocated from the bottom up (from the most junior to the most senior tranche). The senior tranche (often rated AAA) is unaffected, unless the amount of the losses exceeds the amount in the subordinated tranches.

  Other types of internal and external credit enhancement are discussed in https://en.wikipedia.org/wiki/Credit_enhancement.
- **FICO score**. A FICO score is a three-digit number ranging from 300 to 850 (and up to 900 for some industry-specific scores). These scores are largely based on borrower's credit reports (statements generated by the consumer credit reporting bureaus that detail your credit activity and current credit situation) and can help creditors assess how likely the borrower is to repay debt.
- **Cash out refinancing** (in the case of real property) occurs when a loan is taken out on property already owned, and the loan amount is above and beyond the cost of transaction, payoff of existing liens, and related expenses. The difference of cash-out refinancing from equity loans is that cash-out refinancing replaces current mortgage with a new larger mortgage while home equity loan is a new loan in addition to existing mortgage.

Section 2: CMO Structure

A CMO deal consists of three components:

- A legal owner or entity is designated as owning the pool of mortgages.
- That same pool of mortgages serves as collateral, and these mortgages are further subdivided into classes, or tranches, based on the repayment dates of the mortgages.
- Finally, the structure of the agreement describes the way in which money will be distributed to investors

2.1 Loss Allocation Rules

Support against default risk is performed using credit tranching, which is one of the forms of security credit protection. In the simplest case, credit tranching means that any credit losses will be absorbed by the most junior class of bondholders until the principal value of their investment reaches zero. If this occurs, the next class of bonds absorb credit losses, and so forth, until finally the senior bonds begin to experience losses. The subordinated (junior) securities are typically labelled with letter B, mezzanine securities are labelled with letter M, and senior securities are labelled with letter A. Additional credit protection can be added between senior securities and in some cases between different classes of securities.

In general the structure of credit protection is a combination of two rules:

- **Hierarchical structure**, which specifies subordination between CMO securities with respect to allocation of recognized losses from the defaults on underlying mortgage loans;
- **Pro-rata structure**, which assumes equal hierarchical order of the securities and allocates recognized losses to different securities proportionally to the securities outstanding balances.

In a typical CMO deal structure, the hierarchical loss allocation structure is specified by the junior - mezzanine - senior (B-M-A) subordination structure of securities and within the securities labelled with the same letter the losses are allocated on pro-rata basis.

2.2 Principal Allocation Rules

If a security is priced at premium on the issue date, then the coupon rate on the security exceeds the security yield rate equal to the sum of risk free rate and security risk premium. In the case of early prepayment investor may incur losses on the security since investor does not receive the coupon payments, which were expected when the security price premium was estimated. Therefore early prepayment may represent a risk to the investor who expects to receive the return to the investment in the form of coupon payments over an extended period of time.

The principal allocation rules are specified for the CMO deal so that to produce securities with different effective maturity term and amortization schedule. Investor who requires protection against early principal prepayment can purchase then the securities in the CMO deal that have an effective maturity term and amortization schedule that satisfies the investor's needs.

In general, the principal allocation rules can be quite complex (and typically are more complex than the loss allocation rules). Most principal allocation structures can be viewed as a combination of hierarchical structure, pro-rata structure, and weight-based structure. The hierarchical and pro-rata structures were described above. In the weight-based structure, the principal is allocated to different securities based on specific weights that do not depend on the securities outstanding balances. For example, 20% of all repaid

principal amount in a given period is allocated to security A1 and the remaining 80% of repaid principal amount is allocated to other senior tranches on pro-rata basis.

The following principal allocation structures are typical for CMO deals.[2]

- Planned amortization class (**PAC**) tranches. PAC tranches use a mechanism similar to a "sinking fund" to establish a fixed principal payment schedule that directs cash-flow irregularities caused by faster- or slower-than-expected prepayments away from the PAC tranche and toward another "companion" or "support" tranche (see below). With a PAC tranche, the yield, average life, and lockout periods estimated at the time of investment are more likely to remain stable over the life of the security.

  PAC payment schedules are protected by priorities which assure that PAC payments are met first out of principal payments from the underlying mortgage loans. Principal payments in excess of the scheduled payments are diverted to non-PAC tranches in the CMO structure called companion or support tranches because they support the PAC schedules. In other words, at least two bond tranches are active at the same time, a PAC and a companion tranche. When prepayments are minimal, the PAC payments are met first and the companion may have to wait. When prepayments are heavy, the PAC pays only the scheduled amount, and the companion class absorbs the excess.

  PAC tranches are now the most common type of CMO tranche. Because they offer a high degree of investor cash-flow certainty, PAC tranches are usually offered at lower yields.

- Targeted amortization class (**TAC**) tranches. TAC tranches also provide more cash-flow certainty and a fixed principal payment schedule, based on a mechanism similar to a sinking fund, but this certainty applies at only one prepayment rate rather than a range. If prepayments are higher or lower than the defined rate, TAC bondholders may receive more or less principal than the scheduled payment. TAC tranches' actual performance depends on their priority in the CMO structure and whether or not PAC tranches are also present. If PACs are also present, the TAC tranche will have less cash-flow certainty. If no PACs are present, the TAC provides the investor with some protection against accelerated prepayment speeds and early return of principal. The yields on TAC bonds are typically higher than yields on PAC tranches but lower than yields on companion tranches.

- Companion tranches. Every CMO that has PAC or TAC tranches in it will also have companion tranches (sometimes called support bonds), which absorb the prepayment variability that is removed from the PAC and TAC tranches. Once the principal is paid to the active PAC and TAC tranches according to the schedule, the remaining excess or shortfall is reflected in payments to the active companion tranche. The average life of a companion tranche may vary widely, increasing when interest rates rise and decreasing when rates fall. To compensate for this variability, companion tranches offer the potential for higher expected yields when prepayments remain close to the rate assumed at purchase. Similar to Type II and Type III PACs, TAC tranches can serve as companion tranches for PAC tranches. These lower-priority PAC and TAC tranches will in turn have companion tranches further down in the principal payment priority. Companion tranches are often offered for sale to retail investors who want higher income and are willing to take more risk of having their principal returned sooner or later than expected.

- Principal-only (**PO**) securities. Some mortgage securities are created so that investors receive only principal payments generated by the underlying collateral. These Principal-Only (PO) securities may be created directly from mortgage pass-through securities, or they may be tranches in a CMO. In purchasing a PO security, investors pay a price deeply discounted from the face value and ultimately receive the entire face value through scheduled payments and prepayments.

[2] http://www.projectinvested.com/markets-explained/the-various-types-of-cmos/#topic-planned-amortization-class-pac-tranches

- ► Interest-only (**IO**) securities. Separating principal payments to create PO mortgage securities necessarily involves the creation of Interest-Only (IO) securities. CMOs that have PO tranches will therefore also have IO tranches. IO securities are sold at a deep discount to their “notional” principal amount, namely the principal balance used to calculate the amount of interest due. They have no face or par value. As the notional principal amortizes and prepays, the IO cash flow declines.
- ► Residuals. CMOs also contain a “residual” interest tranche, which collects any cash flow remaining from the collateral after the obligations to the other tranches have been met.

Many CMO deals specify a "**crossover**" date or "**senior credit support depletion**" date, which is defined as the date when the balances of all junior and mezzanine securities are reduced to zero. The principal allocation rule for these CMO deals typically switches from original complex rules to a simple pro-rata allocation rule, which applies to all remaining senior securities in the deal. We call the pro-rata rule a "simple" rule since it allows to analyze each senior security independently from all other securities in the deal. The principal prepayment rate, which was estimated for the whole deal based on total deal securities balances and total prepaid balances in each period, applies to each individual security if the prepaid principal is allocated on pro-rata basis.

2.3 Modelling Losses and Principal Allocation Rules

Currently the loss allocation and principal allocation rules are modelled in ac.CMO application independently from each other. This generally imposes a strong restriction on modelled structures CMO as, for example, it does not allow to model the CMO deals with senior credit support depletion date in which the principal allocation rules depend on the total losses and respectively total balances of junior and mezzanine balances. The valuation for these CMO deals can be performed only after the senior credit support depletion date, when the principal allocation rule switched to the pro-rata rule.

The ac.CMO application is also generally not applicable to the CMO deals with multiple security classes, in which the loss and principal allocation in some security classes depend on the allocation in other security classes. The ac.CMO application assumes a single loss and principal repayment rate that is applicable to the modelled CMO deal, while multiple security classes with cross-class allocation rules dependence require to model the loss and principal repayment rate for each security class individually.

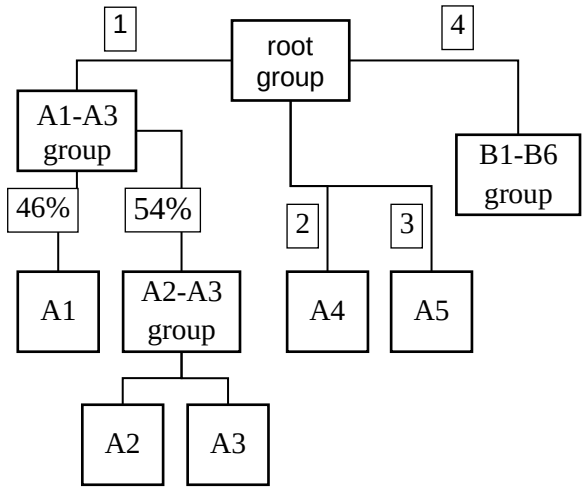
The loss (principal) allocation rule in the ac.CMO application is modelled as a combination of hierarchical, pro-rata, and weight-based structures. The securities in the CMO deal are grouped together depending on which of the three structures is applied to each specific group. The structure is generally modelled as a tree of security groups. Each group can contain either a security or another group of securities. The allocation rule within each group can be different and is specified by one of the above three rules.

To illustrate the modelling approach, suppose that we need to model a simple loss allocation structure in which the losses are allocated based on the subordination structure of the securities. The total group of securities, labelled as G, is divided into three groups: group B of junior securities, group M of mezzanine securities, and group A of senior securities. Within each group B, M, and A the losses are allocated on pro-rata basis. Within group G the losses are allocated using hierarchical structure: first to B, then to M, and finally to A.

2.4 Describing Losses and Principal Allocation Rules

The allocation rules are described in the ac.CMO application using a table that lists all the groups and securities in the CMO deal, the group tree structure, and the allocation rule within each group. An illustration of the table and related group tree are shown in the exhibit below.

Group-tree representation of the loss allocation rules



The table lists all securities and security groups in the first column (the column includes the list of all groups used to describe the allocation structure. In the example, there are three groups used to describe the structure: B1-B6, A1-A3, and A2-A3). The second column specifies respectively each security or security group balance. The third column specifies the parent group of each respective security or security group. Root is the name for the ultimate parent group that contains all securities in the deal. The fourth column describes the hierarchical ranking within the group and the last column describes the weight-based structure within the group. If a specific security or security group in the first column is assigned neither the rank in column four nor the weight in column five, then it is assumed that pro-rata structure is applied to the security (or security group).

The structure can be applied to describe generic loss and principal allocation structures. The ac.CMO application also performs validation of the structure. After the simulation of loss and principal cash flow allocation to each security and each security group, the ac.CMO application performs the following two validation tests:

- The ac.CMO application tests that each parent group is balanced in each period: the parent group balance equals to the sum of its child group balances;
- The ac.CMO application tests that allocation across the children of each parent group is consistent with the allocation rule assigned to the parent group.

Section 3: CMO Valuation

3.1 CMO Cash Flow Model

3.1.1 Overview

The cash flows are estimated using the following sequence of calculations in the ac.CMO application (the cash fluctuations in the application are consistent with the cash flow data analyzed for different actual CMO deals).

1. Set beginning of period $t+1$ security balances equal to the end of period t security balances;
2. Calculate the coupon payments based on the beginning of period security balances;
3. Calculate losses in the balances assuming constant loss rate (estimated based on historical loss data). Adjust balances for the default losses;
4. Calculate mandatory amortization for the outstanding balances. Adjust balances for mandatory amortization;
5. Calculate voluntary prepayment of the outstanding balances. Adjust balances for voluntary prepayment. The adjusted balances are end of period $t+1$ balances

The cash flow model assumes that the *losses are recognized* first and then the mandatory and voluntary balances are estimated based on residual balances.

3.1.2 Cash flow calculations

The cash flows in the CMO securities are calculated by following the steps below.

► Coupon payments, losses in security balances, and voluntary prepayment in security balances are calculated directly as

$$\begin{aligned} C_t &= c_t \times B_t \\ L_t &= \alpha \times B_t \\ X_t^v &= \gamma \times (B_t - L_t) \end{aligned} \tag{3.1}$$

where c_t is coupon rate in period t, and α and γ are fixed loss and voluntary prepayment rates.

► For floating coupon payments calculated as Libor + spread, estimated forward Libor rates based on floating-to-fixed swap curve.[3]

► Mandatory amortization amount X_t^m is estimated as follows.

$$X_t^m = \frac{c \times (B_t - L_t) \times (1+c)^{N-t}}{(1+c)^{N-t} - 1} - c \times (B_t - L_t) = \frac{c \times (B_t - L_t)}{(1+c)^{N-t} - 1} \tag{3.2}$$

(The mandatory amortization is estimated based on the following principle: the payment in each period is fixed and equal to $X = c \times B_t + X_t^m$. The next period balance (assuming zero losses and voluntary prepayment) is equal to $B_{t+1} = B_t - X_t^m = B_t \times (1+c) - X$. The last period balance is equal to

[3] Note that forward Libor rates are different from projected expected Libor rates. Applying forward rates is based on the assumption that investor into the floating-rate security hedges the volatility in Libor rates by entering in Libor forward contracts.

$$B_{t+n} = B_t \times (1+c)^n - X \times (1+c)^{n-1} - \cdots - X = B_t \times (1+c)^n - X \times \frac{(1+c)^n - 1}{c} = 0$$

Therefore the fixed payment is equal to $X = \frac{c \times (B_t - L_t) \times (1+c)^{N-t}}{(1+c)^{N-t}-1}$ and planned amortization is equal to the difference in fixed payment and coupon payment).

The end of period outstanding balances in each security are estimated as follows

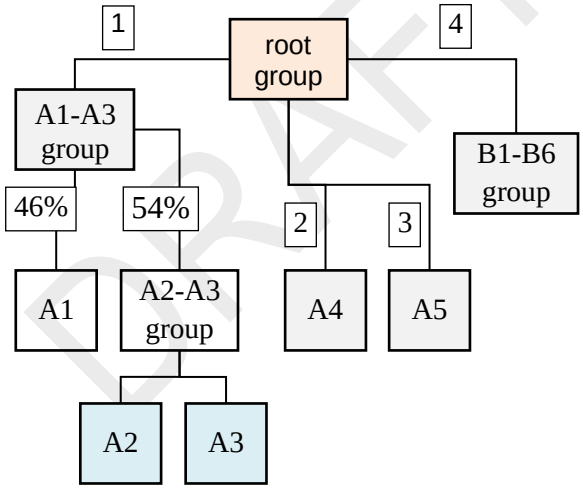
$$B_t = B_{t-1} - L_t - X_t^m - X_t^v \qquad (3.3)$$

- The cash flows are allocated starting from the CMO deal root security group into each child security group and then sequentially from each parent group into each child group, based on the loss and principal repayment allocation rules set in each parent group.

3.1.3 Cash flow allocations across tranches

As discussed above, the CMO tranches are modelled as a tree with the loss/amortization structure defined for the child tranches of each tranche in the tree. The cash flow allocations can be represented schematically by the following diagram.

Group-tree representation of the loss allocation rules



The total loss, amortization, and remaining balances are calculated for the total CMO structure (the root tranche) using equations (3.1) – (3.3). After the cash flows are calculated for the root tranche, the cash flows are calculated recursively for each level of the tranche tree. The cash flows for the second-level tranches are calculated based on the balances of first-level tranches, etc.

It is assumed that the allocation structure for each set of child tranches is either (i) hierarchical, (ii) weighted, or (iii) pro-rata (but not the mix of the above structures). Different sets of child tranches may have different allocation structures. The cash flows are allocated across the child tranches as follows.

- **Hierarchical structure**. Total balance of the parent tranche is allocated first to the child with rank=1, until the balance of the tranche is reduced to zero, then to tranche with rank=2, etc. The allocation is performed in a single iteration.

- **Pro rata structure**. The cash flows are calculated on pro rata basis proportionally to the outstanding child tranche balances. All positive balances remain positive if pro rata allocation rule is applied. The allocation is performed in a single iteration.
- **Weighted structure**. The cash flows are calculated based on respective weights and allocated to the child tranches. If the allocated cash flows exceed the outstanding balances for some of the child tranches, total residual unallocated cash flow is estimated and is allocated (using weighted structure) to the remaining child tranches with positive balances. The allocation may generally require multiple iterations.

3.2 Parameter Estimation

Loss and principal prepayment rates are estimated using total balances, total losses, total repaid principal, and weighted-average effective coupon rate in the CMO deal. (The weighted-average effective coupon rate is applied to estimate the mandatory total principal amortization amount).

3.2.1 Loss Rate Estimation

3.2.1.1 Distribution of Losses

Distribution of loss rates is derived based on the following simple model.

Suppose that there are n mortgages in total and the default on each mortgage is independent across mortgages and across periods. All mortgages are assumed to be equal in size (the size of the average mortgage is denoted as M). In the event of default, fraction $R < 1$ of the mortgage value is recovered from the sale of the house.[4] The loss on a specific mortgage, L_{it}, is a random variable distributed as

$$L_{it} = \begin{cases} (1-R)M, & p \\ 0, & 1-p \end{cases}$$

where p is the probability to default in a given period on a specific mortgage. For a fixed probability p and large enough number of mortgages the total losses converge to the Normal distribution. However, in practice the probability of default p is small. Therefore, we assume that as n increases, the factor $\alpha = p \times n$ stays constant so that the distribution of total loss converges to the Poisson distribution with parameter a. The Poisson distribution is described by the following formula.

$$P_k = e^{-\alpha}\frac{\alpha^k}{k!}$$

In addition, if R is interpreted as the probability of recovery conditional on default, then the distribution L_{it} can be equivalently presented as

$$L_{it} = \begin{cases} M, & p(1-R) \\ 0, & 1-p(1-R) \end{cases}$$

so that the total losses are distributes as Poisson distribution with recovery-adjusted frequency parameter $\tilde{\alpha} = p(1-R) \times n$. The advantage of the loss rate representation with the recovery-adjusted frequency

[44] Note that if the value of the house exceeds the outstanding nominal amount of the mortgage loan, the mortgage holder can either sell or refinance the house and repay the loan. Therefore the borrower will default on the loan only if the value of the house will not cover the remaining loan balances (if $R < 1$).

parameter $\tilde{\alpha}$ of Poisson distribution is that it does not require to estimate parameter R but instead blends parameter R into the parameter $\tilde{\alpha}$ of the Poisson distribution. The total losses in period t are estimated as

$$L_t = \sum_i L_{it} \sim M \times P(\tilde{\alpha})$$

where M is the average size of the outstanding mortgage balances and $B = nM$ represents outstanding total mortgage balances. In the simulations, parameter M is estimated at the issue date as the average mortgage balance and then in each consecutive period the parameter is adjusted so that the ratio $\frac{M_t}{B_t}$ is constant at each period of time (the loss amount is proportional to the current outstanding balances).

3.2.2 Estimation of Loss Parameter

There are two alternative approaches to estimate the loss rate parameter:

1. Direct approach based on Poisson distribution and historical sample of losses in the CMO deal. The ratio of loss to average loan balance, $\frac{L_t}{\frac{1}{n}B_t}$ is estimated for each period. The Poisson distribution with recovery-adjusted Poisson parameter $\tilde{\alpha}$ is estimated based on the constructed sample of loss rates;
2. Indirect approach based on security public market prices. Security are traded in the market. The prices of the securities are either directly reported or estimated for example by Bloomberg. Loss parameter generally has a one-sided impact on the security prices: the higher the loss rate, the lower is the price of each security. Therefore there is a single loss rate that minimizes the sum of squared between the actual observed security prices and the prices estimated using NPV valuation approach based on the cash flow model.

3.2.3 Principal Prepayment Rate Estimation

The principal payments consist of two separate parts:

1. Mandatory principal amortization. Each period a mortgage borrower pays a fixed amount so that the mortgage loan principal is repaid completely on the maturity date. The condition determines the fixed payment made by the borrower in each period, which includes the coupon payment for the period and amortized principal amount. The mandatory principal amortization is calculate as the difference between the estimated fixed monthly payment and the coupon payment.
2. In addition to mandatory principal amortization, the mortgage loan borrower has an option to prepay in part or the whole mortgage loan. The voluntary principal prepayment depends in general on multiple economic and borrower-specific factors and is modelled as a random process. The prepayment rate determines how quickly the outstanding balances are reduced in each security and, therefore, represent a significant factor in the valuation of the security risks.

Note that in ac.CMO tool the balances are adjusted first for the losses and then for the mandatory principal amortization prior to principal prepayment rate estimation.

The prepayment rate generally does not have a one-sided impact on the security prices. A higher prepayment rate may have both a positive or negative effect on the security prices and the effect may be different for different securities. The impact depends on the CMO deal structure and on the estimated loss rate. With high loss rate, high prepayment rate may result in lower losses and as a result in higher security price. With low loss rate, high prepayment rate may result in lower effective maturity term and as a result in lower security price.

Because of the uncertain impact of the prepayment rate on security prices, the recommended approach is to estimate the prepayment rate parameter based on the historical sample of prepayment rates in the CMO deal.

3.2.4 PSA Prepayment model

The PSA Prepayment Model is a prepayment scale developed by the Public Securities Association in 1985 for analyzing American mortgage-backed securities. The PSA model assumes increasing prepayment rates for the first 30 months after mortgage origination and a constant prepayment rate thereafter. This approximates real-world experience that during the first few years, mortgage borrowers:

- ► are less likely to relocate to a different home,
- ► are less likely to refinance into a new mortgage, and
- ► are less likely to make extra payments of principal.

The standard model (also called "100% PSA") works as follows: Starting with an annualized prepayment rate of 0.2% in month 1, the rate increases by 0.2% each month, until it reaches 6% in month 30. From the 30th month onward, the model assumes an annualized prepayment rate of 6% of the remaining balance. Each monthly prepayment is assumed to represent full payoff of individual loans, rather than a partial prepayment that leaves a loan with a reduced principal balance.

Variations of the model are expressed in percent; e.g., "150% PSA" means a monthly increase of 0.3% in the annualized prepayment rate, until the peak of 9% is reached after 30 months. The months thereafter have a constant annualized prepayment rate of 9%.

3.3 Summary of CMO Valuation Model

CMO securities valuation is performed by following the steps below.

1. Based on CMO prospectus, estimate the security structure:
   - ► Summarize the list of securities in the CMO deal;
   - ► Summarize the outstanding balances of the securities;
   - ► Summarize the coupon payment on the securities;
   - ► Summarize loss allocation rules and construct the respective security group tree and related ac.CMO tool input table;
   - ► Summarize principal allocation rules and construct the respective security group tree and related ac.CMO tool input table;
2. Download security-specific data on losses, principal payments, outstanding balances, and coupon payments. Validate consistency of the historical data against the allocation rules summarized above;
3. Download available market prices for the traded securities in the CMO deal;
4. Estimate mandatory principal amortization for the total balances of the CMO deal;
5. Estimate the historical loss rate based on either (i) historical loss rate data or on (ii) security market prices;
6. Estimate the historical voluntary principal prepayment rate based on historical voluntary prepayment rate data;

7. Estimate the cash flows in each security of the CMO deal based on the estimated parameters and formulas described above;
8. Estimate the risk-free discount rates applicable to the CMO securities.
   - ► Estimate risk-free yield term structure (based for example on Treasury rates);
   - ► Convert the yield curves into the related risk-free discount factors D_t^*.
9. **Deterministic approach**. Estimate the price of each security as the NPV of the related security cash flows. For the CDS contracts that hedge the CMO securities, estimate the NPV of losses in CDS-covered tranches and NPV of the fixed periodic payments to the CDS seller. Estimate the CDS price as the difference between the two NPV values;
10. **Monte-Carlo simulation approach.**
    - ► Construct samples of loss rate and principal prepayment rate based on the estimated distributions of the loss rate and principal prepayment rate. If the loss rate is estimated based on the market prices of the securities, then construct the sample of principal prepayment rates only;
    - ► Construct the sample of security prices (CDS prices) for each given loss rate and principal prepayment rate as described in step 9.
    - ► Estimate the price of each security in the CMO deal as the average of the sample security prices.

Note that the Monte-Carlo approach takes into account not only the average loss and principal prepayment rate in the security valuation, but also variation in the rates. Therefore the Monte-Carlo approach is generally more accurate and more robust for security price valuation purposes

Section 4: Hedging CMO Securities with CDS Contracts

4.1 Overview

The credit default swap provides an additional credit support structure for a given CMO security. The support is structured as follows. The senior security (“A”) is divided into three tranches: junior (“A.J”), mezzanine (“A.M” or “A.CDS”), and senior (“A.S”). The balance of the A.J tranche is typically selected to match the price discount at which security A was acquired. From accounting perspective, the losses in the A.J tranche are covered by the security A price discount (since the total value of the security is recorded in the balance sheet at the purchase price and not at the nominal value). The losses in the A.CDS tranche are covered by the CDS contract. The purpose of the A.J and A.CDS tranches is to provide additional credit support for the A.S tranche. All losses in A securities are first accumulated in A.J and then in A.CDS tranches.

The sizes of the A.J and A.M sub-tranches are determined using “attachment point” and “detachment point”. For example, if attachment point equals 5% and detachment point equals 15%, then the size of the A.J tranche is equal to 5% of the A security size and size of the A.M tranche is equal to 10% (10% = 15% - 5%) of the A security. Note that over time the attachment and detachment points change with the change in the losses and prepaid amounts.

The order of principal payments is specified by the master agreement that provides the terms of the CDS instruments. A standard assumption is that the tranche A.S is repaid first, then, after the tranche A.S balance is reduced to zero, tranche A.CDS is repaid. Tranche A.J is repaid last.

The price of the CDS instrument depends on the distribution of losses in the A.CDS sub-tranche of the A security. The key parameters that affect the distribution of losses in the A.M tranche are the following.

1. The outstanding balances and attachment and detachment points of security A;
2. Loss and principal allocation structure of the CMO deal (which determines the loss and principal allocation to the A security);
3. Parameters of loss and principal repayment distributions estimated for the total pool of underlying mortgage loans;

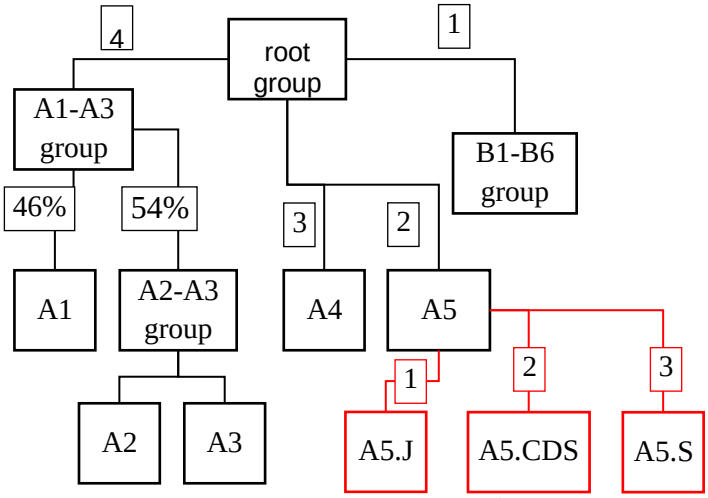
Each CDS contract can be equivalently interpreted as an insurance policy underwritten on the CMO security. The attachment point is interpreted as the insurance deductible and the detachment point is interpreted as the insurance maximum limit.

Alternatively, the CDS contract can be interpreted as a portfolio of long call option with strike equal to attachment point and short call option with a strike equal to detachment point, which is underwritten for the CMO security (the portfolio is also known as a bear spread).

4.2 CDS Contract Valuation

CDS contract can be modelled as follows. First the tranching of the security is incorporated as a part of the CMO deal structure. For example, suppose that security A5 in the example above was tranched into junior, CDS and senior sub-loans. Then the modified CMO deal structure would be represented using the following tree.

Group-tree representation of the loss allocation rules (including CDS protection structure)



The modelled allocation of losses and principal and interest payments includes now securities (tranches) A5.J, A5.CDS, and A5.S. Note that by definition

$$NPV\ L + NPV\ I^{rf} + NPV\ P = Par$$

(The net present value of the security losses L, principal payments P and risk free interest payments I^{rf} equal to par value, which is by definition equal to the security nominal amount and accrued interest). The value of the security V is equal to

$$NPV\ C + NPV\ P = V$$

where C is security coupon payments. Therefore,

$$NPV\ L - NPV\ (C - I^{rf}) = Par - V$$

where $C - I^{rf}$ can be interpreted as the risk spread paid on the security. Based on the above equation, if the CDS contract compensates all losses in the CDS tranche and receives the risk spread on the CDS tranche as a periodic payment, then the fixed price of the CDS contract is equal to the difference in par value and the contract price estimated using NPV valuation approach (this is the equation used by JP Morgan Chase for CDS contract valuation).

In practice the periodic payment on a CDS contract C^{CDS} may be equal to zero, risk spread, coupon rate, or other selected value. A general equation for the CDS price based on NPV valuation is

$$V^{CDS} = NPV\ L - NPV\ C^{CDS}$$

Appendix A: Bloomberg CMO Data and Functions

Bloomberg data and functions are illustrated for the BOAMS 2007-2 CMO deal and BOAMS 2007-2 A13 tranche, which structure is presented in detail in Appendix D.1.

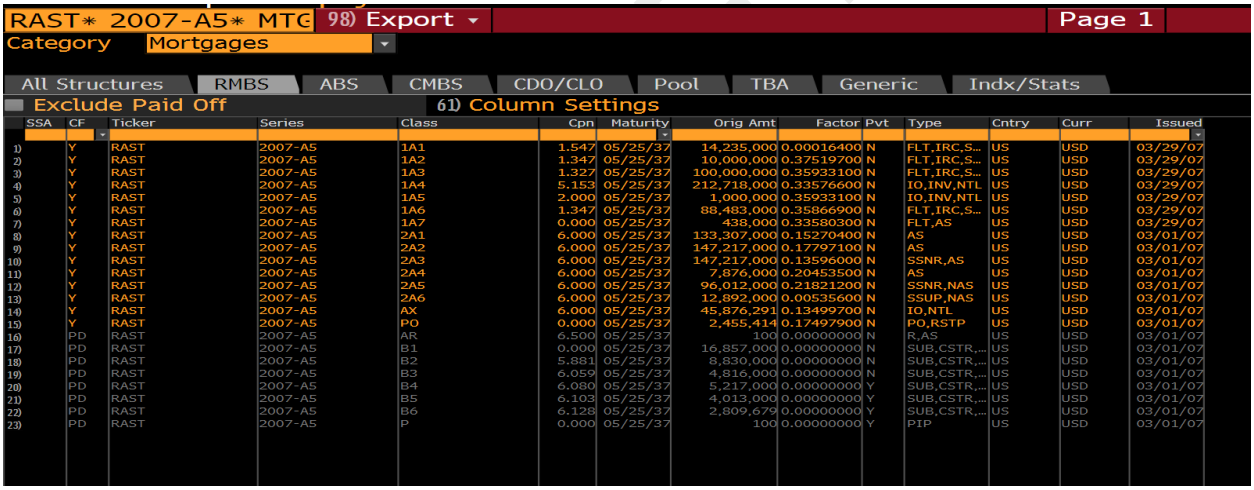
A.1 Bloomberg notations

The following notations and terminology is sued by Bloomberg.

A.2 CMO tranche structure

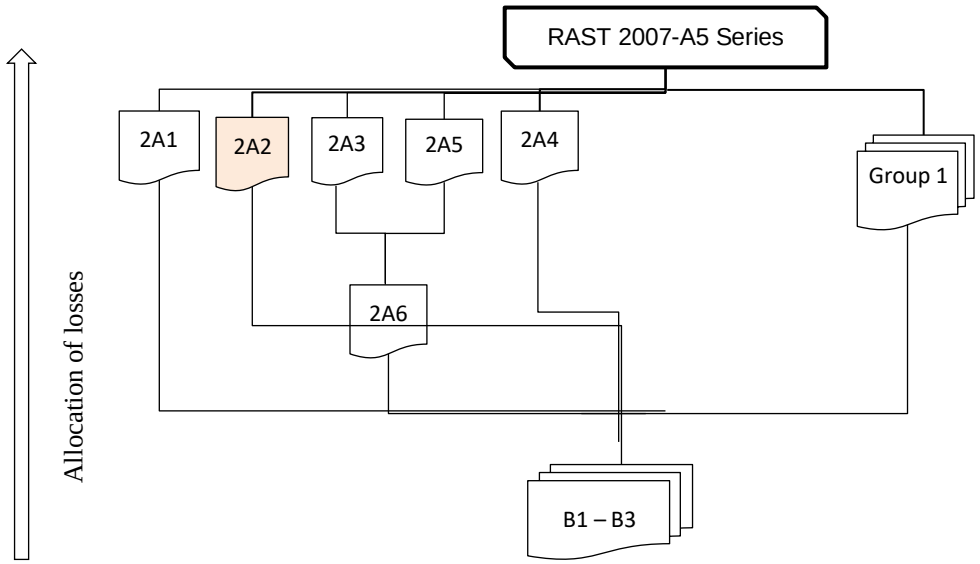
To display the CMO deal structure, type the name of the CMO deal.

Alternatively, you can type a specific tranche ticker; remove from the ticker the tranche name and click enter.



	SSA	CF	Ticker	Series	Class	Cpn	Maturity	Orig Amt	Factor	Pvt	Type	Cntry	Curr	Issued
1)		Y	RAST	2007-A5	1A1	1.547	05/25/37	14,235,000	0.00016400	N	FLT,IRC,S...	US	USD	03/29/07
2)		Y	RAST	2007-A5	1A2	1.347	05/25/37	10,000,000	0.37519700	N	FLT,IRC,S...	US	USD	03/29/07
3)		Y	RAST	2007-A5	1A3	1.327	05/25/37	100,000,000	0.35933100	N	FLT,IRC,S...	US	USD	03/29/07
4)		Y	RAST	2007-A5	1A4	5.153	05/25/37	212,718,000	0.33576600	N	IO,INV,NTL	US	USD	03/29/07
5)		Y	RAST	2007-A5	1A5	2.000	05/25/37	1,000,000	0.35933100	N	IO,INV,NTL	US	USD	03/29/07
6)		Y	RAST	2007-A5	1A6	1.347	05/25/37	88,483,000	0.35866900	N	FLT,IRC,S...	US	USD	03/29/07
7)		Y	RAST	2007-A5	1A7	0.000	05/25/37	438,000	0.33580300	N	FLT,AS	US	USD	03/29/07
8)		Y	RAST	2007-A5	2A1	6.000	05/25/37	133,307,000	0.15270400	N	AS	US	USD	03/01/07
9)		Y	RAST	2007-A5	2A2	6.000	05/25/37	147,217,000	0.17797100	N	AS	US	USD	03/01/07
10)		Y	RAST	2007-A5	2A3	6.000	05/25/37	147,217,000	0.13596000	N	SSNR,AS	US	USD	03/01/07
11)		Y	RAST	2007-A5	2A4	6.000	05/25/37	7,876,000	0.20453500	N	AS	US	USD	03/01/07
12)		Y	RAST	2007-A5	2A5	6.000	05/25/37	96,012,000	0.21821200	N	SSNR,NAS	US	USD	03/01/07
13)		Y	RAST	2007-A5	2A6	6.000	05/25/37	12,892,000	0.00535600	N	SSUP,NAS	US	USD	03/01/07
14)		Y	RAST	2007-A5	AX	6.000	05/25/37	45,876,291	0.13499700	N	IO,NTL	US	USD	03/01/07
15)		Y	RAST	2007-A5	PO	0.000	05/25/37	2,455,414	0.17497900	N	PO,RSTP	US	USD	03/01/07
16)		PD	RAST	2007-A5	AR	6.500	05/25/37	100	0.00000000	N	R,AS	US	USD	03/01/07
17)		PD	RAST	2007-A5	B1	0.000	05/25/37	16,857,000	0.00000000	N	SUB,CSTR,...	US	USD	03/01/07
18)		PD	RAST	2007-A5	B2	5.881	05/25/37	8,830,000	0.00000000	N	SUB,CSTR,...	US	USD	03/01/07
19)		PD	RAST	2007-A5	B3	6.059	05/25/37	4,816,000	0.00000000	N	SUB,CSTR,...	US	USD	03/01/07
20)		PD	RAST	2007-A5	B4	6.080	05/25/37	5,217,000	0.00000000	Y	SUB,CSTR,...	US	USD	03/01/07
21)		PD	RAST	2007-A5	B5	6.103	05/25/37	4,013,000	0.00000000	Y	SUB,CSTR,...	US	USD	03/01/07
22)		PD	RAST	2007-A5	B6	6.128	05/25/37	2,809,679	0.00000000	Y	SUB,CSTR,...	US	USD	03/01/07
23)		PD	RAST	2007-A5	P	0.000	05/25/37	100	0.00000000	Y	PIP	US	USD	03/01/07

An example below illustrates that the CMO deal consists of two groups of senior loans and a group of subordinated loans. The class description can be used to check correctness of the class structure (which should be estimated from the deal prospectus). The structure of the above deal is illustrated I the diagram below.



The description of classes shows that classes 2A3 and 2A5 are super senior (SSNR) and class 2A6 is a senior support class (SSUP), which is consistent with the above diagram.

To check credit support depletion date, enter the ticker name of the most senior tranche in subordinated and mezzanine group of tranches and check its balances.

RAST 2007-A5 B1 Mtge | Export | Paydown Information

US RMBS 3.646(204)157 CUSIP 76114HAS4 Loan Level

All Collateral

Summary | Paydown | Collateral | Performance

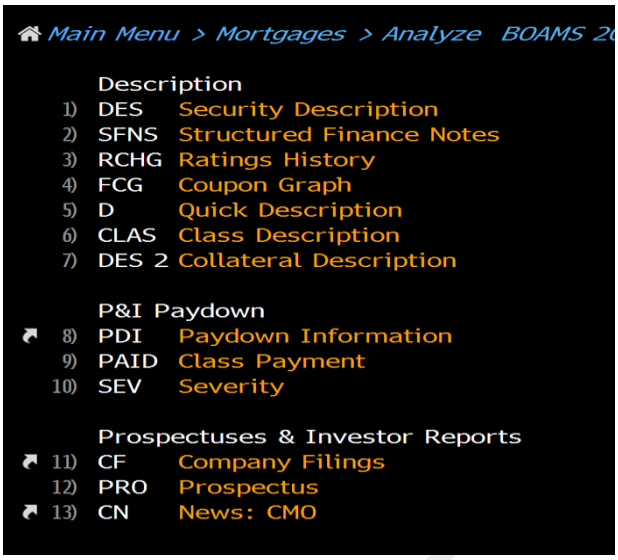
Tranche SUB,CSTR,NAS Coupon 0 WALA 157 Issue 03/01/2007
WAC 3.6461 WAM 204 Maturity 05/25/2037
Orig Bal 16,857,000 Day Count 30/360 Pay Delay 24 days

	Date	Factor	Coupon	Principal	Losses	Interest	Balance	Ca
	Total			183,785	16,673,215	2,279,175		2,4
14)	10/25/2010	0.000000000	2.43947	0.00	0.00	0.00	0.00	
15)	09/27/2010	0.000000000	2.43947	0.00	0.00	0.00	0.00	
16)	08/25/2010	0.000000000	2.43947	0.00	0.00	0.00	0.00	
17)	07/26/2010	0.000000000	2.43947	0.00	0.00	0.00	0.00	
18)	06/25/2010	0.000000000	2.43947	0.00	0.00	0.00	0.00	
19)	05/25/2010	0.000000000	2.43947	0.00	0.00	0.00	0.00	
20)	04/26/2010	0.000000000	2.43947	0.00	1,766,590.53	0.00	0.00	
21)	03/25/2010	0.104798631	4.56297	0.00	1,962,411.53	0.00	1,766,590.53	
22)	02/25/2010	0.221213861	5.20863	0.00	2,371,665.51	400.76	3,729,002.06	
23)	01/25/2010	0.361907075	5.55661	0.00	3,200,593.15	2,500.64	6,100,667.57	2
24)	12/28/2009	0.551774380	5.70577	0.00	2,185,388.23	18,661.63	9,301,260.72	18
25)	11/25/2009	0.681417153	5.88089	0.00	5,186,566.22	25,206.87	11,486,648.95	25
26)	10/26/2009	0.989097418	5.95516	0.00	0.00	38,322.51	16,673,215.17	38
27)	09/25/2009	0.989097418	5.98447	0.00	0.00	42,045.34	16,673,215.17	42

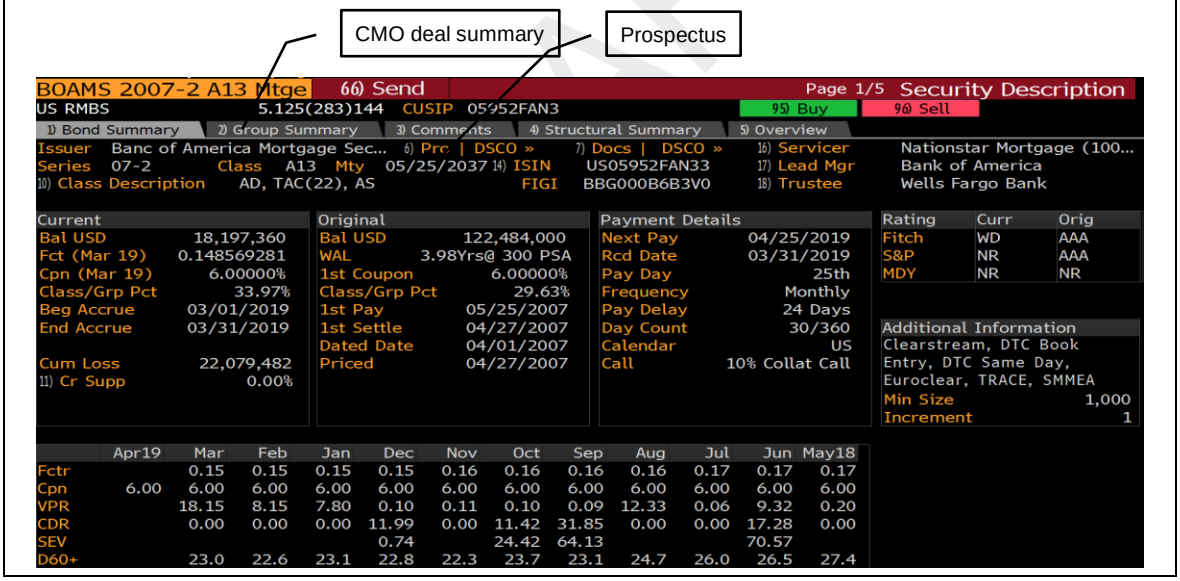
In the exam[le, the credit support depletion date is April 2010.

A.3 CMO tranche data

CMO Tranche Description

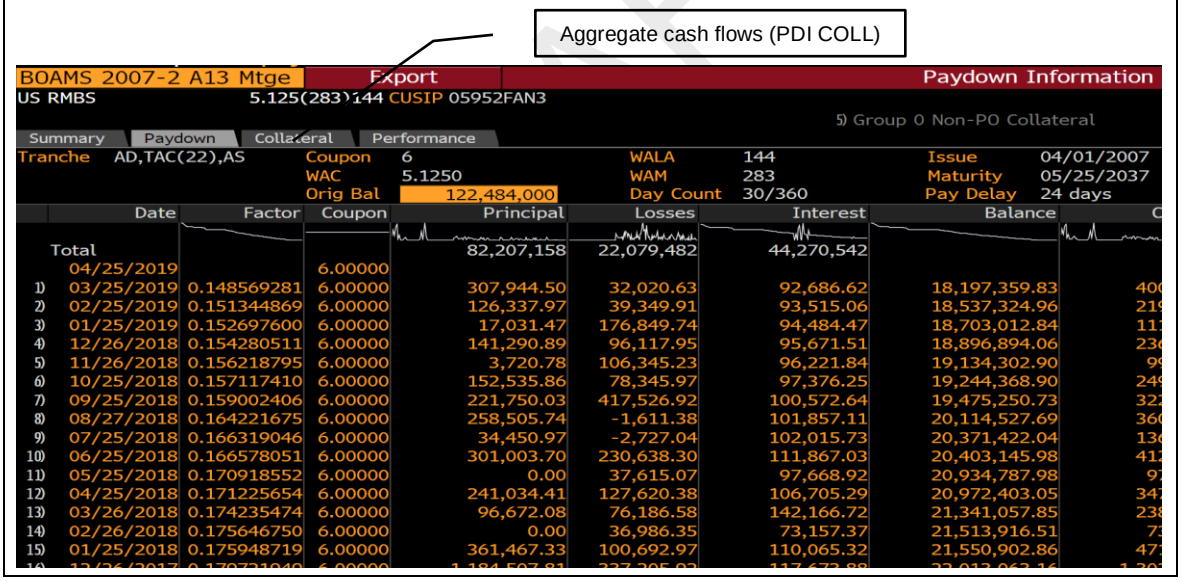


CMO Tranche Summary



CMO Tranche Overview

CMO Tranche Historical Payments

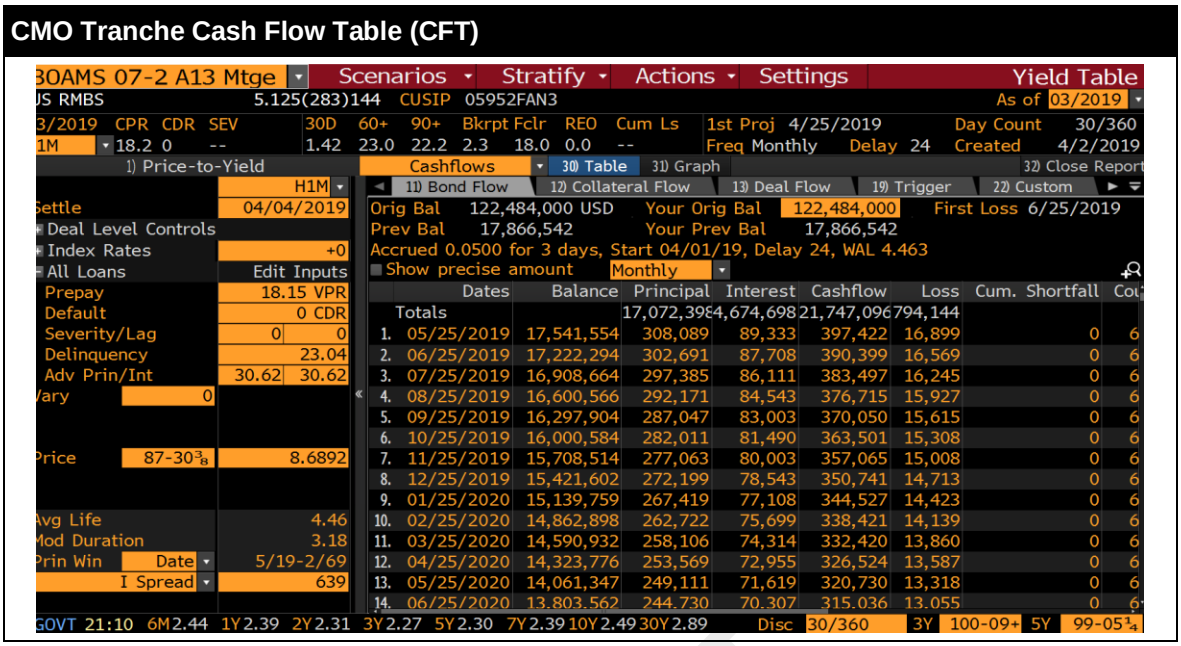


A.4 CMO tranche valuation

The valuation is performed either using CFT or BVAL functions.

Bloomberg **CFT** function output.

CMO Tranche Cash Flow Table (CFT)



Bloomberg **BVAL** function output

A.5 Abbreviations

Bloomberg uses the following abbreviations.

- CDR – constant default rate
- CPR – conditional prepayment rate
- VPR – voluntary prepayment rate
- Severity – the ratio of the liquidation loss and recovery percentage
- WAC – weighted average coupon

- ► WAM – weighted average maturity
- ► WALA – weighted average loan age
- ► Credit Score
- ► HPI LTV – home price indexed amortized loan to value

A.6 Bloomberg functions

List of Bloomberg functions used to retrieve CMO securities data is provided below.

- ► CPD:
- ► DPD:
- ► LLD:
- ► PRO: CMO deal prospectus;
- ► SEV:
- ► HP (GP): historical prices as a table (or as a graph)

Appendix B: Bloomberg CMO Valuation Model

Appendix C: ac.finance.CMO Tool

The objective of the ac.CMO tool is to model the structure of interest/principal/loss distributions in a CMO deal and estimate projected allocations in each CMO tranche.

C.1 Modelling CMO structure

The overall set of tranches is converted to a tree set, where each node of the tree is either an individual tranche or a group of tranches. A tranche (group of tranches) is a child of a group of tranches if it belongs (is a subset) of the group. The potential implemented allocation structures are limited to the following types:

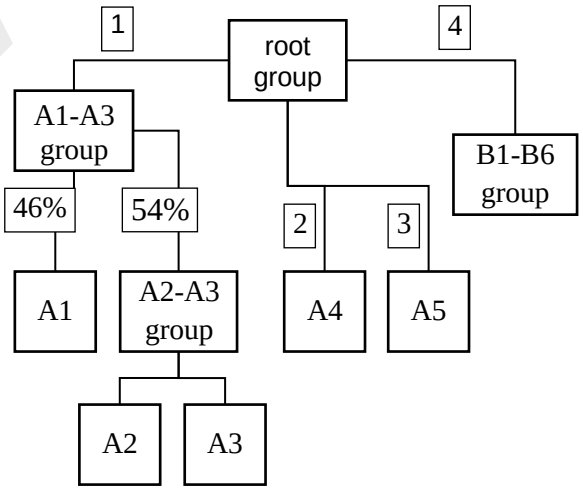
- ► Hierarchical distributions, which is described by the integer ranking index. The allocation is made first to the tranches with higher ranking index;
- ► Weighted distributions, which are described by respective weight of total principal/loss amount allocated to each tranche or group of tranches.
- ► Pro rata distribution, which is a default distribution and is assumed whenever neither rank nor weight is specified for the node.

In the example below, losses are distributed first to tranches B1-B6, then to tranche A4, next to tranches A1-A3, and finally to tranche A5 (hierarchical structure). Within the A1-A3 group, 46% of the loss is distributed to tranche A1 and remaining 54% is distributed to tranches A2-A3 (weighted distribution). Finally the loss distributions between A2 and A3 tranches are made on the pro rata basis.

Table representation of the loss allocation rules

Tranche Name	Outstanding Balance	Loss Parent	Loss Rank	Loss Weight
B1-B6	20,000,000	root	4	
A1	50,000,000	A1-A3		46.0%
A2	20,000,000	A2-A3		
A3	60,000,000	A2-A3		
A4	100,000,000	root	3	
A5	20,000,000	root	1	
A1-A3	130,000,000	root	2	
A2-A3	80,000,000	A1-A3		54.0%

Group-tree representation of the loss allocation rules



The ac.CMO tool allows to model the structures, which are arbitrary composition of the hierarchical, weighted, and pro rata distribution structures. The ac.CMO tool however is not able currently to model the structures that switch between different distribution rules (and therefore the tool cannot model structures with a crossover point. The tool can be applied to evaluate the deals after the crossover date).

C.2 CMO valuation

The following Excel functions are available for CMO valuation.

1. `ac.finance.cmo.model`: create a CMO model (including loss and amortization tree models). The function has the following arguments:
   - Name of the CMO deal;
   - Fixed or floating coupon rate;
   - Number of modelled periods;
   - Table with the description of the loss and amortization ranking structure.
2. `ac.finance.cmo.calc`: create a CMO calculator object. The function has the following arguments:
   - CMO object;
   - List of discount factors for different periods;
   - Loss rate mean;
   - Loss rate standard deviation;
   - Amortization rate mean;
   - Amortization rate standard deviation.
   - Generated sample size of CMO objects.
3. `ac.finance.cmo.fn`: create a CMO function. The function is created for the purpose of implied parameters estimation. The function has the following arguments:
   - CMO calculator object;
   - Parameters mapping. Parameters include the following key=.value pairs.
     - argument-keys => List of argument keys (which included loss-rate, amortization-rate, and coupon-rate keys);
     - value-tranche-names => List of matched tranche name prices.

   The function uses a subset of (loss rate, amortization rate and coupon rate) as arguments depending on the provided argument keys. The returned value is the list prices estimated for the tranches specified by the 'value-tranche-names' key.

C.3 Validation of CMO valuation results

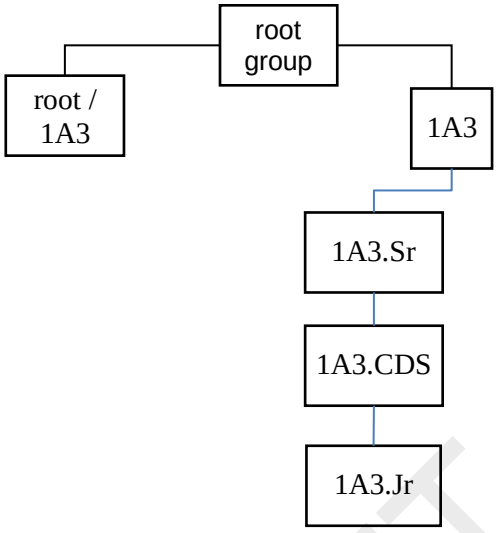
The validation of the modelling results can be performed using (i) the validation output built in the tool; and (ii) the output of simplified CMO valuation model.

C.3.1 Built-in validation output

The tool has built-on check of model structure consistency and validation of the model cash flow balances. The generated tree structure of the model can be displayed as the tool output and reviewed for consistency with the modelled structure. An illustration of the tree structure is presented in the diagram below,

Loss ranking structure		*Amortization ranking structure*	
1A3[pr]	1A3.Jr	1A3[pr]	1A3.Sr
	1A3.CDS		1A3.CDS
	1A3.Sr		1A3.Jr
root/1A3[pr]		root/1A3[pr]	

The output was produced for the following structure. The losses / amortization are allocated on pro-rata basis to all classes. The modelled class 1A3 has a hierarchical ranking structure of it 1A3.Sr, 1A3.CDS, and 1A3.Jr sub-classes.



The ranking structure of the CMO classes is presented both for (I) loss allocation; and for (ii) allocation of amortized principal balances. The [pr] abbreviation implies pro-rata allocation of the loss/amortization balances.

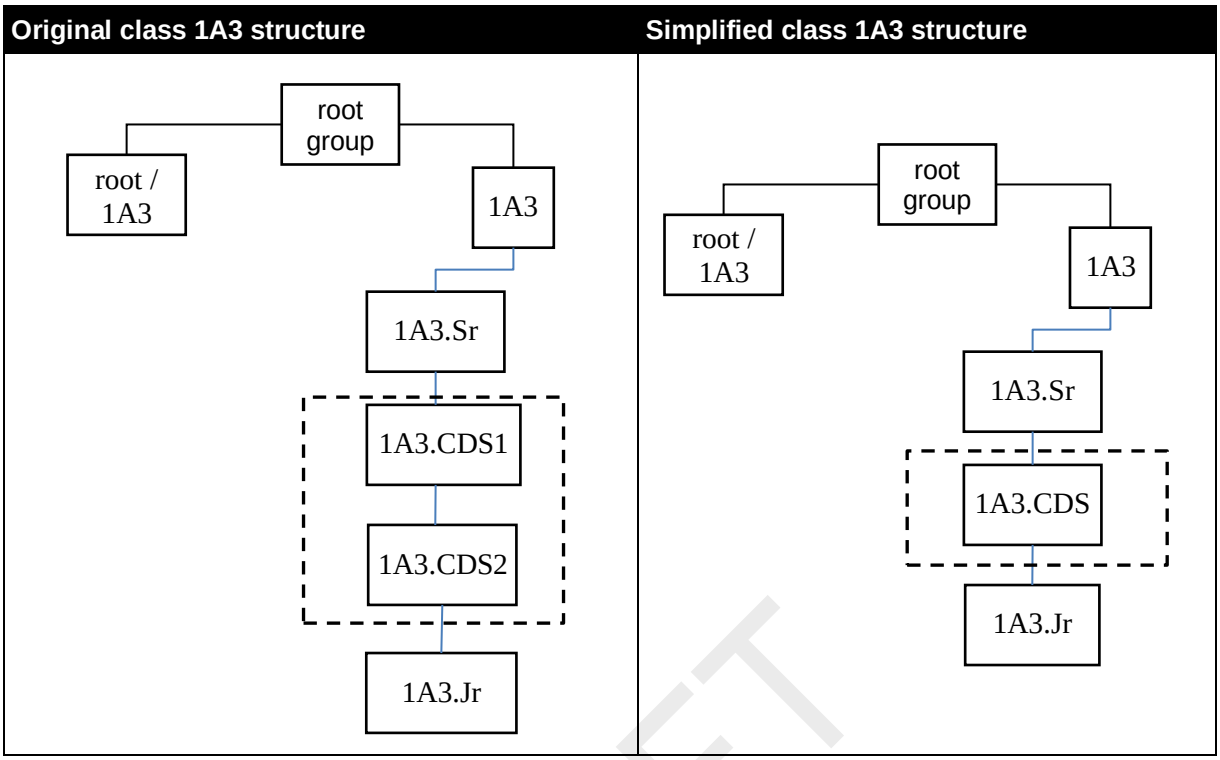
The balance validation procedure verifies that generated CMO class balances are consistent with the CMO class structure. If consistency is violated, a message is displayed which reports for which class, which cash flow type, and in which period the calculated balance is inconsistent with the structure. If no messages are displayed, then no inconsistencies were identified.

C.3.2 Validation based on simplified modelling assumptions

Reasonability of produced results can also be reviewed by simplifying the CMO structure and reviewing the valuation results for the simplified model. The approach is illustrated for the following example. Suppose that modelled class 1A3 has multiple CDS sub-tranches, which were implemented to protect the losses in the 1A3 class. Then the valuation of the CDS classes can be broken down into two sub problems:

1. The valuation of the CDS sub-tranches aggregated into a single CDS tranche;
2. Break-down of the price estimated for the aggregate CDS sub-tranche into the prices estimated for each individual CDS sub-tranche.

The first step verifies that the overall price assigned to the CDS sub-tranches of the modelled class is consistent with the tested price (implying that the allocation between the CDS buyer and seller is reasonable). The second step verifies that not only the total allocation for CDS protection is reasonable but also each individual CDS price is reasonable.



The approach allows to better track a potential source of the tested CDS prices inconsistencies.

C.3.3 Comparative statics analysis

Appendix D: An Approximate Approach to CDS Pricing

In this section, we describe a high-level approach that does not require modelling the credit enhancement / principal prepayment structure of CMO loan tranches and does not require running Monte-Carlo simulations to assess the value of the CDS instruments. The valuation approach is based on the assumption that the expectation of the losses in A tranche can be estimated from the Bloomberg reported price of the tranche and that the standard deviation of the losses is typically relatively small and not sensitive to modelling parameters. Basically, the resulting distribution of losses that determine the prices of the A4.J, A4.M, and A4.S sub-tranches is relatively insensitive to the modelling approach and modelling parameters given that the price of the A tranche is fixed.

Below we provide a simplified model of CDS pricing. Suppose that the A tranche is modelled as amortized loan where the loss L and the monthly amortized amount X are fixed and constant each month. Losses start to be recognized starting from date T^L. Similarly, the principal amortization starts from period T^A. The values of T^L and T^A for each tranche depend on credit support and principal amortization support balances as well as on the loss and amortization rates. By T^{min} and T^{max} we denote $T^{min} = min(T^L, T^A)$ and $T^{max} = max(T^L, T^A)$.

Suppose that i represents the coupon rate and y represents the risk-free discount rate. If original balance is B_0 then outstanding balance in period $t \geq T^{max}$ is $B_t = B_{T^{max}} - (t - T^{max})(L + X)$. The balance is reduced to zero ($B_T = 0$) in period $T = T^{max} + \frac{B_{T^{max}}}{L+X}$. The NPV value[5] of the amortized principle amounts is

$$P^X = \sum_t \frac{X}{(1+y)^t} = \frac{1}{(1+y)^{T^{max}}} \left[\frac{X}{y}\left(1 - \frac{1}{(1+y)^{T-T^{max}}}\right)\right].$$

Coupon payment in period t is $I_t = iB_t = i\big(B_0 - t(L+X)\big)$. The NPV of the coupon payments is

$$P^I = \frac{iB_0}{y}\left(1 - \frac{1}{(1+y)^T}\right) - \frac{i(L+X)}{y}\left(\frac{1+y}{y}\left(1 - \left(\frac{1}{1+y}\right)^T\right) - T\left(\frac{1}{1+y}\right)^T\right)$$

The price of the amortized loan is estimated as

$$P = \frac{X + iB_0}{y}\left(1 - \frac{1}{(1+y)^T}\right) - \frac{i(L+X)}{y}\left(\frac{1+y}{y}\left(1 - \left(\frac{1}{1+y}\right)^T\right) - T\left(\frac{1}{1+y}\right)^T\right)$$

Together with the equation

$$T = \frac{B_0}{L+X}$$

we can estimate the values of X and L and derive the expected losses in the A tranche and, after making certain assumptions on the standard deviation of the losses, we can derive the price of the sub-tranches of the A tranche. Substituting out the $L + X$ in the second equation, we obtain the following equation for the amortized amount X:

[5] The simplest generic approach to estimate different discounted sums expressions is via the function $\varphi(x) = \sum_{t=1,\dots,T} \frac{x^t}{(1+y)^t} = \frac{x}{1+y-x}\left(1 - \left(\frac{x}{1+y}\right)^T\right)$. Using the function formula, we can estimate that $\sum_{t=1,\dots,T} \frac{1}{(1+y)^t} = \varphi(1) = \frac{1}{y}\left(1 - \left(\frac{1}{1+y}\right)^T\right)$, and $\sum_{t=1,\dots,T} \frac{t}{(1+y)^t} = \varphi'^{(1)} = \frac{1+y-x-x(-1)}{(1+y-x)^2} \times \left(1 - \left(\frac{x}{1+y}\right)^T\right) - T\frac{x}{1+y-x} \times \frac{1}{1+y} \times \left(\frac{x}{1+y}\right)^{T-1} = \frac{1+y}{y^2}\left(1 - \left(\frac{1}{1+y}\right)^T\right) - T\frac{1}{y}\left(\frac{1}{1+y}\right)^T$

$$P = \frac{X + iB_0}{y}\left(1 - \frac{1}{(1+y)^T}\right) - \frac{\frac{iB_0}{T}}{y}\left(\frac{1+y}{y}\left(1 - \left(\frac{1}{1+y}\right)^T\right) - T\left(\frac{1}{1+y}\right)^T\right)$$

or

$$X = Py + \frac{iB_0}{T} \times \left(\frac{1+y}{y}\left(1 - \left(\frac{1}{1+y}\right)^T\right) - T\left(\frac{1}{1+y}\right)^T\right) - iB_0 \times \left(1 - \frac{1}{(1+y)^T}\right)$$

After simplifying we obtain

$$X = Py + \frac{iB_0}{T}\left(\frac{1+y}{y}\left(1 - \left(\frac{1}{1+y}\right)^T\right) - T\right) \tag{1.1}$$

and

$$L = \frac{B_0}{T} - X \tag{1.2}$$

In the above equations, the number of periods T is measured in months, the coupon rate i and risk free discount rate y are monthly rates.

Appendix E: Examples

E.1 Description of a CMO deal structure and statistics

For illustrative purposes, we show how the valuation is performed for the BOAMS 2007-2 CMO deal (Banc of America Mortgage Securities, Inc. Mortgage Pass-Through Certificates, Series 2007-2). The CMO deal is described as follows.

- Prospectus link and the link to Excel working file with the CMO deal historical cash flows.
- The deal was issued in April 2007 and original total nominal principal amount mortgage pool loans was US$416,134,218. As of Sep 2017, the remaining balance of the CMO deal was US$72,295,042. The balances of junior B1-B3 and mezzanine M1-M2 securities were exhausted in August 2011;
- CMO deal security structure. The deal has one class of senior securities A1 to A14. The security balances are summarized in the exhibit below.

Security class	Nominal balance amount				Allocation shares		Coupon rate
	Issue date		September 2017		principal	Loss	
	in US$	in %	in US$	in %	In %	In %	In %
Classes B1 to B3	6,732,000	1.62%	0				
Classes M1 to M2	9,678,000	2.33%	0				
Class A1	40,001,000	9.61%	7,266,267	10.05%	10.10%	10.15%	1.24%
Class A3	37,306,000	8.96%	10,940,373	15.13%	15.21%	15.29%	6.13%
Class A4	480,000	0.12%	0				
Class A5	69,246,000	16.64%	565,527	0.78%	0.79%	0.79%	5.62%
Class A6	20,000,000	4.81%	2,722,129	3.77%	3.78%	3.80%	5.87%
Class A7	49,528,000	11.90%	17,972,954	24.86%	24.99%	25.12%	5.62%
Class A9	25,656,000	6.17%	0				
Class A10	9,812,500	2.36%	2,848,599	3.94%	3.96%	3.98%	5.97%
Class A11	9,812,500	2.36%	2,877,618	3.98%	4.00%	4.02%	6.28%
Class A12	6,413,000	1.54%	1,861,714	2.58%	2.59%	2.60%	6.13%
Class A13	122,484,000	29.43%	24,204,957	33.48%	33.65%	33.82%	6.13%
Class A14	1,550,000	0.37%	0				
Class 30-PO	7,435,148	1.79%	1,034,904	1.43%			
Total	**416,134,148**	**100%**	**72,295,042**	**100%**	**100%**	**100%**	

A2 and A8 are interest only (IO) securities. A9 security is composed of three tranches: A-9-1, A-9-2 and A-9-3. A1 is a floating rate security. Classes A3 and A13 have 6.0% fixed coupon rate, Classes A5 and A7 have 5.5% fixed coupon rate, class A6 has 5.75% fixed coupon rate, and classes A10-A12 have on average 6.0% fixed coupon rate. (The estimated effective coupon rate, presented in the last column, is higher by approximately 13bps for each class).

The CMO deal has a *credit support depletion date* after which the complex initial principal allocation rules switch to simple pro-rata allocation rules. The credit support depletion date became effective on *25 August 2011*. The pro-rata allocation rules are validated in the above exhibit. The percentage of allocated total losses and principal balances to each security approximately equals to the percentage of the security outstanding balances relative to the deal total outstanding balances. The allocation of losses and principal amount in one month prior and one month after credit support depletion date are presented in the exhibit below.

Certificate class	July 2011				September 2011			
	Nominal balance		principal	loss	Nominal balance		principal	loss
	in US$	in %	In %	In %	in US$	in %	In %	In %
Classes B1 to B3								
Classes M1 to M2	752,053	0.30%	0.00%	99.97%				
Class A1	25,029,936	9.98%	10.17%	0.00%	24,233,024	10.01%	9.79%	10.17%
Class A3	37,306,000	14.87%	0.00%	0.00%	36,300,621	15.00%	14.59%	0.00%
Class A4	480,000	0.19%	0.00%	0.00%	234,363	0.10%	0.19%	19.34%
Class A5	1,655,847	0.66%	66.39%	0.00%	1,611,223	0.67%	0.65%	0.00%
Class A6	8,907,853	3.55%	10.89%	0.00%	8,667,790	3.58%	3.48%	0.00%
Class A7	49,528,000	19.74%	0.00%	0.00%	48,193,244	19.91%	19.38%	0.00%
Class A9	13,144,210	5.24%	12.29%	0.00%	12,431,941	5.14%	5.14%	29.75%
Class A10	9,812,500	3.91%	0.00%	0.00%	9,500,086	3.93%	3.84%	3.99%
Class A11	9,812,500	3.91%	0.00%	0.00%	9,548,058	3.94%	3.84%	0.00%
Class A12	6,413,000	2.56%	0.00%	0.00%	6,208,821	2.57%	2.51%	2.61%
Class A13	83,378,232	33.23%	0.00%	0.00%	80,723,606	33.35%	32.62%	33.88%
Class A14	0							
Class 30-PO	4,675,237	1.86%			4,385,340	1.81%		
Total	**250,895,368**	**100%**	**100%**	**100%**	**242,038,117**	**100%**	**100%**	**100%**

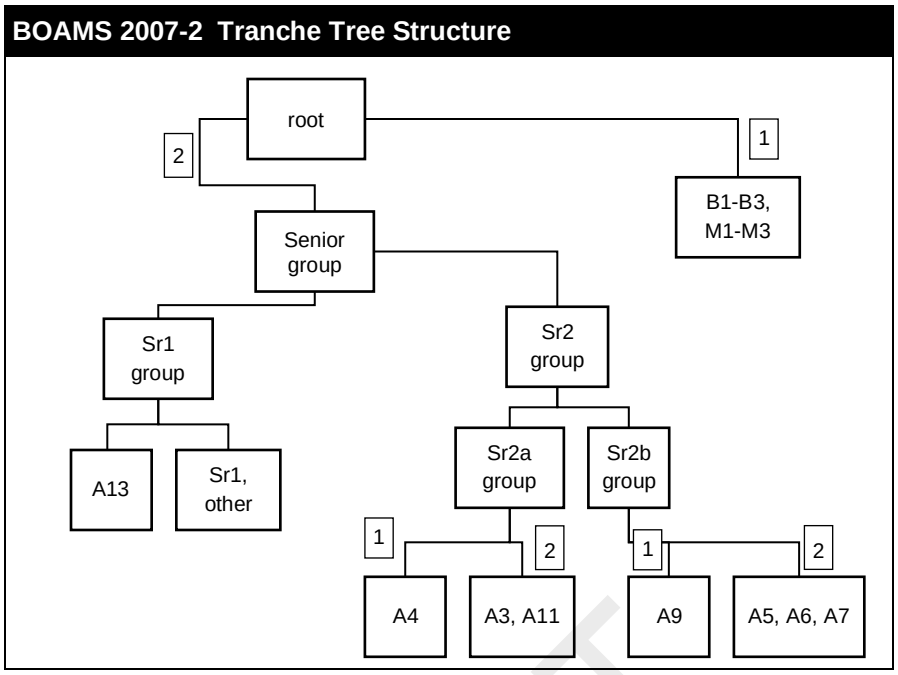
In addition to credit support from junior and mezzanine tranches, there is credit support between senior tranches. Super senior certificates A3 and A11 are supported by class A4 and super senior certificates A5, A6 and A7 are supported respectively by classes A9-1, A9-2 and A9-3.

- Loss allocation structure. As showed in the exhibits above, the losses are allocated on pro-rata basis to securities A1, A10, A12 and A13 after the credit support depletion date. The losses are allocated on pro-rata basis to all securities after November 2014 when the balances of all super senior support securities (which include A4 and A9 classes) are reduced to zero;
- Principal allocation structure. As showed in the exhibit above, the principal payments are allocated on pro-rata basis to all securities in the deal after the credit support depletion date. Prior to the credit support depletion date the principal allocation rules are more complex with the largest share of repaid principal amount being allocated to class A5;

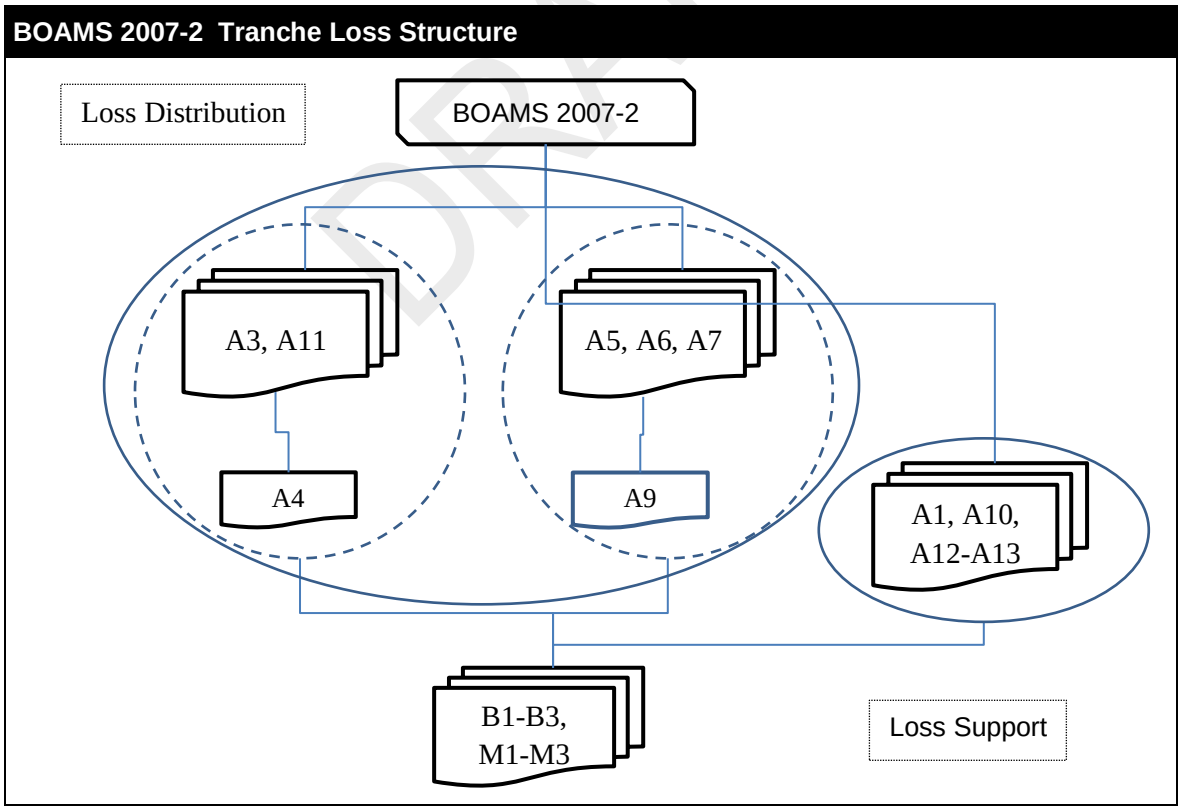
The structure is described by the following input table in the ac.CMO tool.

Tranche Name	Outstanding Balance	Loss Parent	Loss Rank	Loss Weight	Amortization Parent	Amortization Rank	Amortization Weight
B1-B3,M1-M3	0	root	1		root		
Senior	245,468,077	root	2		root		
Sr1	124,633,668	Senior			Senior		
A13	83,378,232	Sr1			Sr1		
Sr1,other	41,255,436	Sr1			Sr1		
Sr2	120,834,409	Senior			Senior		
Sr2a	47,598,500	Sr2			Sr2		
A4	480,000	Sr2a	1		Sr2a		
A3,A11	47,118,500	Sr2a	2		Sr2a		
Sr2b	73,235,909	Sr2			Sr2		
A9	13,144,210	Sr2b	1		Sr2b		
A5,A6,A7	60,091,700	Sr2b	2		Sr2b		

The tranche tree structure is represented as follows.



The tree structure also shows the ranking of tranches with respect to loss distribution. If no ranking is shown, the losses are distributed on pro rata between the group child tranches. Equivalently the loss structure can be described by the following diagram.



E.2 Bloomberg data

After the structure of the CMO deal is summarized, the data for individual classes of the CMO deal is retrieved from Bloomberg. The class cash flows are obtained using the CPD function and exported to Excel. The Excel data output is illustrated in the diagram below.

Date	Factor	Coupon	Pass-through Rate	Principal	Losses	Interest	Balance	Cashflow
Total				103,539,614	17,099,427	62,968,680		166,508,294
03/25/2020		6.00						
02/25/2020	0.18	6.00		18,798	-18,366	132,892	26,577,959	151,690
01/27/2020	0.18	6.00		222,840	-11,038	133,951	26,578,391	356,791
12/26/2019	0.18	6.00		248,339	25	135,193	26,790,193	383,532
11/25/2019	0.18	6.00		208,826	153,018	137,002	27,038,557	345,828
10/25/2019	0.19	6.00		237,565	-4,898	138,165	27,400,401	375,731
09/25/2019	0.19	6.00		126,252	0	138,797	27,633,068	265,048
08/26/2019	0.19	6.00		648,726	-5,542	142,013	27,759,320	790,738
07/25/2019	0.19	6.00		371,831	-6,960	158,460	28,402,503	530,291
06/25/2019	0.20	6.00		0	1,733	129,222	28,767,374	129,222
05/28/2019	0.20	6.00		534,532	210,673	164,455	28,769,107	698,987
04/25/2019	0.20	6.00		0	16,752	130,772	29,514,312	130,772

The cash flows include the monthly interest distributed to the CMO class, loan principal repayment (which included both voluntary and mandatory principal repayment), losses incurred by the class, and outstanding balance as of the months end. Factor column shows the percentage of the outstanding balance relative to the original class balance. Cashflow column reports the sum of interest and repaid principal amounts.

The cash flows table allows to track the losses and amortized principal balances for each period of time. The new period class outstanding balance is calculated as the previous period class outstanding balance minus amortized principal balance and minus the incurred losses. Negative losses imply that certain portion of the recorded losses was reversed.

In addition to the individual class cash flows, the table reports the total cash flows for the CMO deal. The total cash flows are reported at a more detailed break-down. Specifically, the losses include as components (i) forgiveness; (ii) liquidation; (iii) forbearance; and (iv) other. The amortized principal components include (i) scheduled principal amortization and (ii) unscheduled principal amortization.

The cash flow data can be used to validate the structure of the CMO deal and to validate certain assumptions of the model. For example, if the losses and principal amortization balances are allocated to all classes on pro rata basis, then the losses / amortized principal allocated to each specific class are equal to respective share of the class outstanding balance. The scheduled / unscheduled principal amortization data can be used to validate the principal amortization equations of the model.

E.3 Simple numeric example of CMO security credit and prepayment risk

Consider the following stylized example.

- A CMO security has a balance of $100 million and pays fixed 6% interest rate. The expected remaining maturity term of the security is 10 years;

- ► The losses are accumulated in the security at 1% rate per year;
- ► The principal balances are prepaid in the security at the rate 2% per year;

Exercise 1. Estimate the NPV of the security;

Exercise 2. Suppose that there is a 10% probability that 10% of the security balance is lost in year one. Re-estimate the NPV of the security.

Exercise 3. Suppose that there is a 10% probability that 20% of the security is prepaid in year one. Re-estimate the NPV of the security.