Essays on Matching
N-lateral K-decision Matching Model
and
Matching Model with Coordination Frictions

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Standard Matching Model

- Classes: Males Females
- Types: (X_1, μ_1) (X_2, μ_2)

$$X_n = \{x_n : x_n = (\text{age, edu, income, preferences}, \ldots)\}$$

- Public good decisions:

$$\mathcal{K} = \{k : k = (\text{work, kids}, \ldots)\}$$

- Preferences: $u_1 = b^k C_1 + f_1(x_1, x_2, k)$
  $u_2 = b^k C_2 + f_2(x_1, x_2, k)$
- Constraint: $C_1 + C_2 \leq g(x_1, x_2, k)$

Stable Matching

- Match Surplus:

$$u(x_1, x_2)$$

- Matching Function

$$x_2 = m(x_1)$$

- Transfers in a Stable Matching

$$\begin{aligned} p_1(x_1) + p_2(x_2) &\geq u(x_1, x_2) \\ p_1(x_1) + p_2(m(x_1)) &= u(x_1, m(x_1)) \end{aligned}$$

Essay one: $\mathcal{M}_N^{\mathcal{K}}$ Model

- $f_n(x_1, x_2, k)$ and $g(x_1, x_2, k)$ - linear in types
- Match Surplus:

$$u^k(x_1, x_2) = \sum_{n=1}^{2} a_n^k x_n + a_0^k \qquad u(x_1, x_2) = \max_k u^k(x_1, x_2)$$

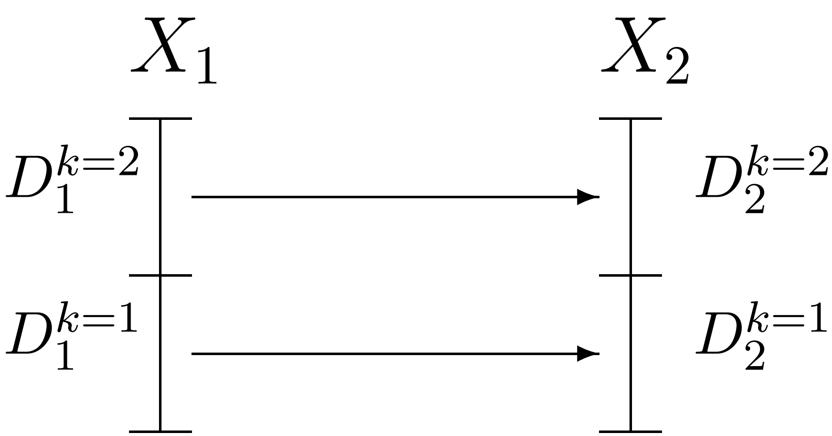
- Transfers:

$$\begin{aligned} p_n(x_n) &= \max_k \left[a_n^k x_n + p_{n,0}^k\right] \\ \sum_{n=1}^{2} p_{n,0}^k &= a_0^k \end{aligned}$$

Equilibrium

Demand sets

$$D_n^k(p) = \{x_n : a_n^k x_n + p_{n,0}^k \geq a_n^l x_n + p_{n,0}^l \quad \text{for any } l \neq k\}$$



1. mass balance: for each k and for any $n, \widetilde{n}$

$$\mu_n(D_n^k) = \mu_{\widetilde{n}}(D_{\widetilde{n}}^k)$$

2. surplus balance: for each k

$$\sum_n p_{n,0}^k = a_0^k$$

Contributions

1.

standard	$\mathcal{M}_N^{\mathcal{K}}$
general f_n, g	linear f_n, g
$p_n(x_n)$	$p_n(x_n) = \max_k \left[a_n^k x_n + p_{n,0}^k\right]$
$m(\cdot) : X_1 \to X_2$?	$\left\{D_n^k\right\}$

2. Existence and uniqueness of equilibrium

3. Matching in groups of N individuals

Essay two: Assortative Matching

Types: $X_1 \subset \mathbb{R}$ $X_2 \subset \mathbb{R}$

- Literature:

$$u(x_1, x_2)$$

  Types are complementary $\Rightarrow$ monotone matching is efficient

- My result:

$$u(x_1, x_2, k)$$

  Types are complementary to each other and types are complementary to decision $\Rightarrow$ monotone matching is efficient

Essay two: Preferences with Idiosyncratic Component

- Types:

$$X_1 = \{1, \ldots, I\} \quad X_2 = \{1, \ldots, J\}$$

- Decisions:

$$\mathcal{K} = \{1, \ldots, K\}$$

- Preferences:

$$\widetilde{u}_{ij}^k = u_{ij}^k + p_{ij}^k + \varepsilon_{ij}^k$$

Contributions

- Introduce Random Preferences in Matching Model
- Existence and Uniqueness of Equilibrium

Essay three: Matching with Frictions

1. Competition between auctions

2. Micro foundations for aggregate matching function in the labor market

Matching with Frictions: Game Description

1. Agents (workers): $i = 1, \ldots, I$
2. Types: $g \in \{1, \ldots, G\}$
3. State, signal: $\mathbf{g} = (g_1, \ldots, g_I) \quad s_i(\mathbf{g}) = g_i$
4. Beliefs (symmetric): $f = (f_1, \ldots, f_G)$
5. Actions (auctions/firms): $m \in \{1, \ldots, M\}$
6. Preferences: Vickrey mechanism at each auction m

Matching with Frictions: Results

- Finite number of types of heterogeneous agents
- Arbitrary number of potentially matched agents at each auction

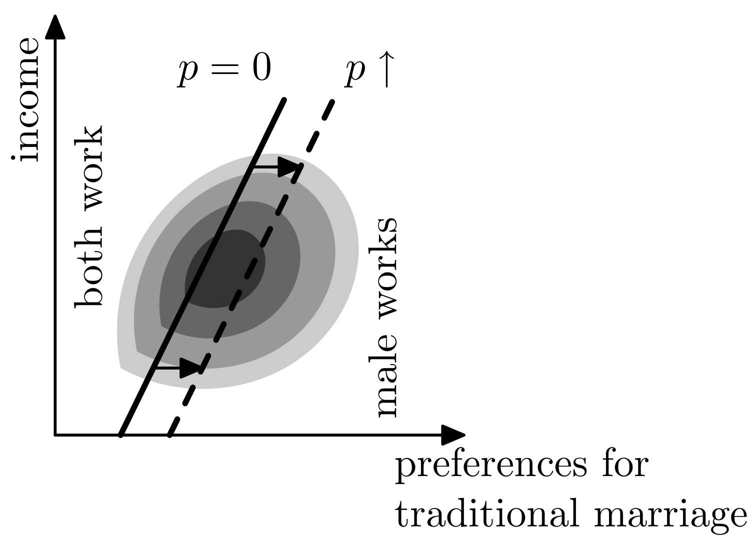
1. Symmetric equilibria of the game $=$ solutions of the planner's constrained optimization problem

2. Concavity of the planner's constrained optimization problem $\Rightarrow$ use numeric methods to construct equilibrium

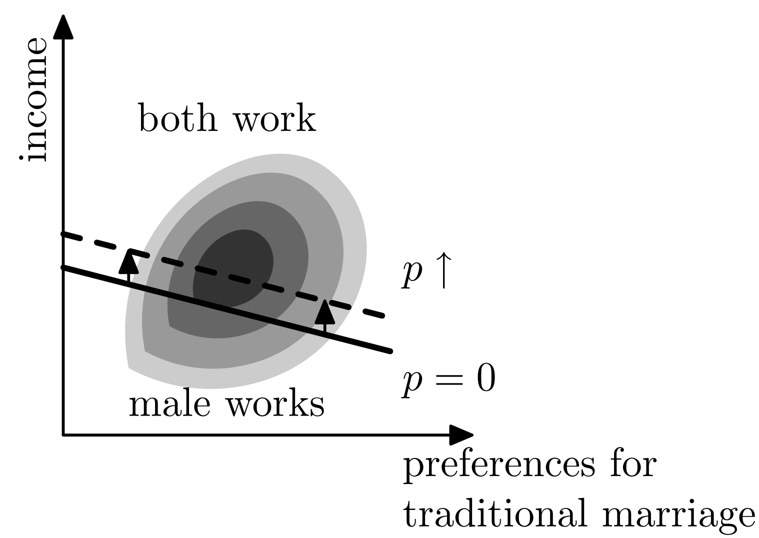
Example

Decisions: $\mathcal{K} = \{$only male works, both work$\}$

Male Types



Female Types



Picture: Surplus Function

$$W^{\text{asym}}(p,q) = 1 + p + q - 2pq \qquad W(p) = 1 + 2p - 2p^2$$